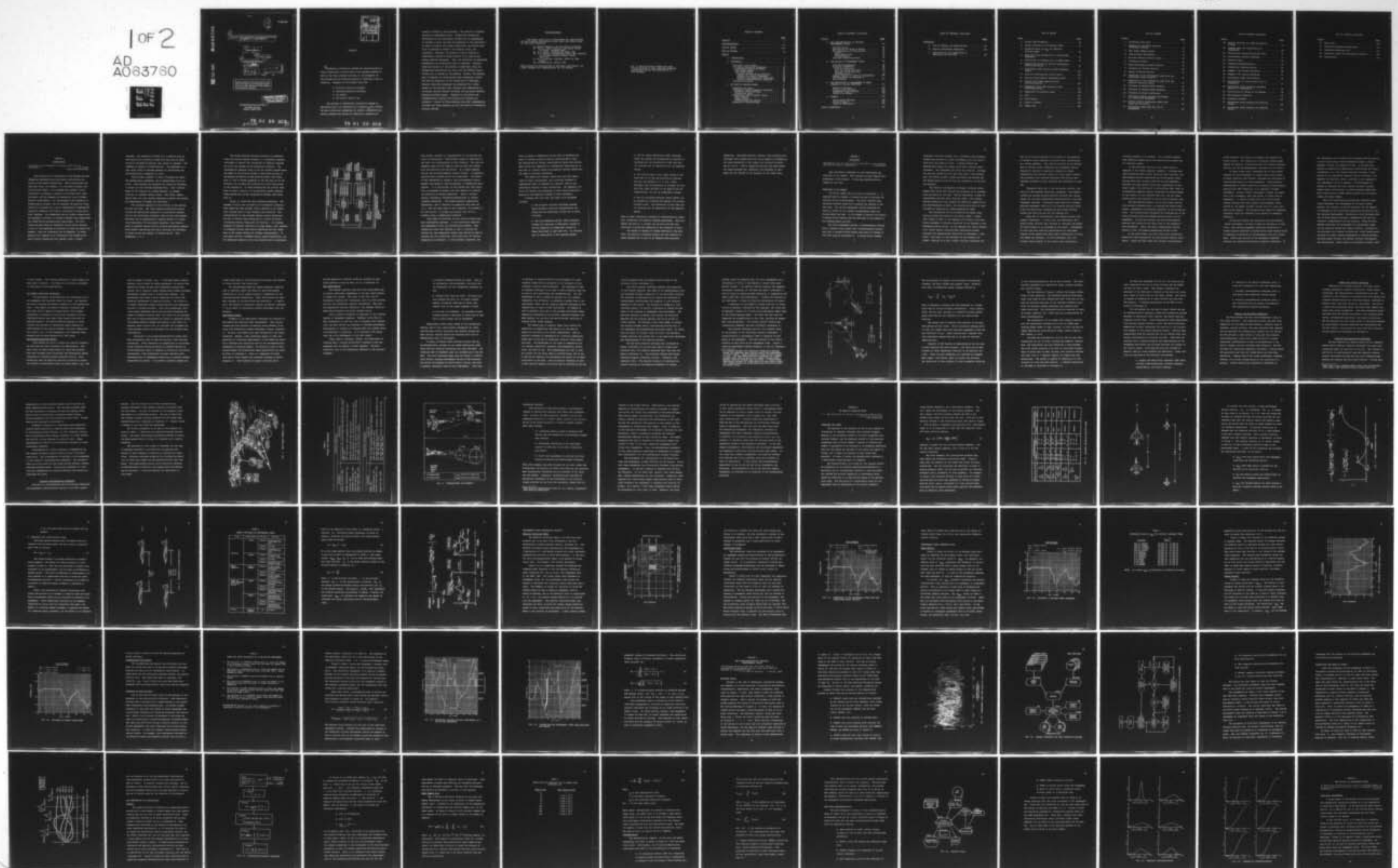
AD-A063 760 CENTER FOR NAVAL ANALYSES ARLINGTON VA F/G 15/3
THE ANALYSIS OF DYNAMICALLY INTERACTIVE SYSTEMS (AIR COMBAT BY --ETC(U)
DEC 78 W R SIMPSON

UNCLASSIFIED NL

1 OF 2
AD
A063760



55 000242.00

2

NW

AD A063760

6

THE ANALYSIS OF DYNAMICALLY INTERACTIVE SYSTEMS (AIR COMBAT BY THE NUMBERS)

10

William R. Simpson

14 CNA-PP

9 Professional Paper No. 242

11 December 1978

12 162p.

DDC
RECEIVED
JAN 26 1979
A

DDC FILE COPY

The ideas expressed in this paper are those of the author. The paper does not necessarily represent the views of either the Center for Naval Analyses or the Department of Defense.

DISTRIBUTION STATEMENT A
Approved for public release;
Distribution Unlimited

CENTER FOR NAVAL ANALYSES
1401 Wilson Boulevard
Arlington, Virginia 22209

79 01 26 033

Ø47 270

ACCESSION for
NTIS White Section
DDC Buff Section
UNANNOUNCED
JUSTIFICATION
Letter on file
BY PP-240
DISTRIBUTION/AVAILABILITY CODES
Dist. AVAIL. and/or SPECIAL
A

ABSTRACT

Dynamically interactive systems are characterized by a mutual interaction in which each of the systems responds to each of the other systems according to its perception of the problem and its perceived methods of achieving a goal or objective. Examples of the problem class are:

- Aircraft collision avoidance
- Destroyer/Submarine Encounter
- Air Combat
- The Child's Game of Tag

The analysis of dynamically interactive systems is approached from a non-deterministic viewpoint. This relaxes the more traditional assumptions of perfect information and perfect response and allows for adaptation responses not

79 01 26 033

normally included in such analyses. The analysis is applied directly to experimental data. Traditional statistical descriptors of the interaction outcomes may be supplemented by statements about the form and substance of the interaction. In order to analyze the system interaction, the problem must first be defined in terms of its history, goals, and priorities. Secondly, the process of merit ordering is undertaken to define a figure-of-merit to be used as the primary analysis parameter. Next, the definition of applicable mathematical and statistical tools is required. Detailed numerical analysis of the problem is undertaken using the mathematical and statistical tools, and the defined figure-of-merit as it relates to the problem. Finally, the development of measures of effectiveness which adequately reflect the form and substance of the interaction is undertaken. Detailed numerical procedures are presented for a given example of the problem class, together with comparisons to previously derived analysis variables and previously defined measures-of-effectiveness. Examples of conclusions and recommendations together with sensitivity analyses are presented. Certain of these measures have been independantly verified, but other measures are not available by alternative analysis methods.

ACKNOWLEDGEMENTS

The author would like to acknowledge the contributions of the following individuals without whom this effort could not have been completed:

Dr. Gerald Gregorek, The Ohio State University
Dr. Phil E. Depoy, Director, OEG Group, Center for Naval Analyses (CNA)
Dr. R.A. Oberle, Professional Staff, CNA
Mr. W.S. Stewart, Professional Staff, CNA (formerly of the Naval Weapons Center, China, Lake, CA)
Mr. M.T. Pilletere, Engineer, Naval Air Test Center
Ms. Durinda Suttle, Typist, CNA

While noting the contributions of the above individuals, the author assumes full responsibility for its content.

To: E. Marie and Paula Simpson who have sacrificed much of their time and tolerated a great deal during the preparations of this document.

TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

TABLE OF CONTENTS (Continued)

LIST OF TABLES

LIST OF FIGURES

LIST OF FIGURES (Continued)

LIST OF FIGURES (Concluded)

CHAPTER 1

INTRODUCTION

"A little bit of competence is worth a whole lot of confidence."

-- R. Jenson
(Texas A&M University)

The problem of the interaction of two systems has been adequately approached only when severe restrictions are placed on the problem formulation. These restrictions may take many forms; for example, in a two-body collision (the billiard ball model), it is assumed that system A (a billiard ball) is unaware of system B (A billiard ball) until collision, and the response is governed by a set of deterministic requirements. Or, the assumption that system A is unaware may be relaxed, and its response can again be formulated but as governed by a set of deterministic responses to system B's presence, and finally a collision (or near collision) response. The assumptions can be further relaxed until the response of system A to system B becomes very complicated and indeed may be mathematically unsolveable. The overriding and most decisive assumption in the entire analysis is that if the mechanism of encounter is known (no matter how complex), then the interaction can be computed. In short, system B will respond to an interaction with system A and every similar interaction with system A with a "given"

response. The response of system B is a function only of the state of the universe at some time zero and its interaction is governed by physical laws (known or unknown). The response of system B is deterministic. This approach to a very large class of problems presents no difficulties and the deterministic assumption is valid.

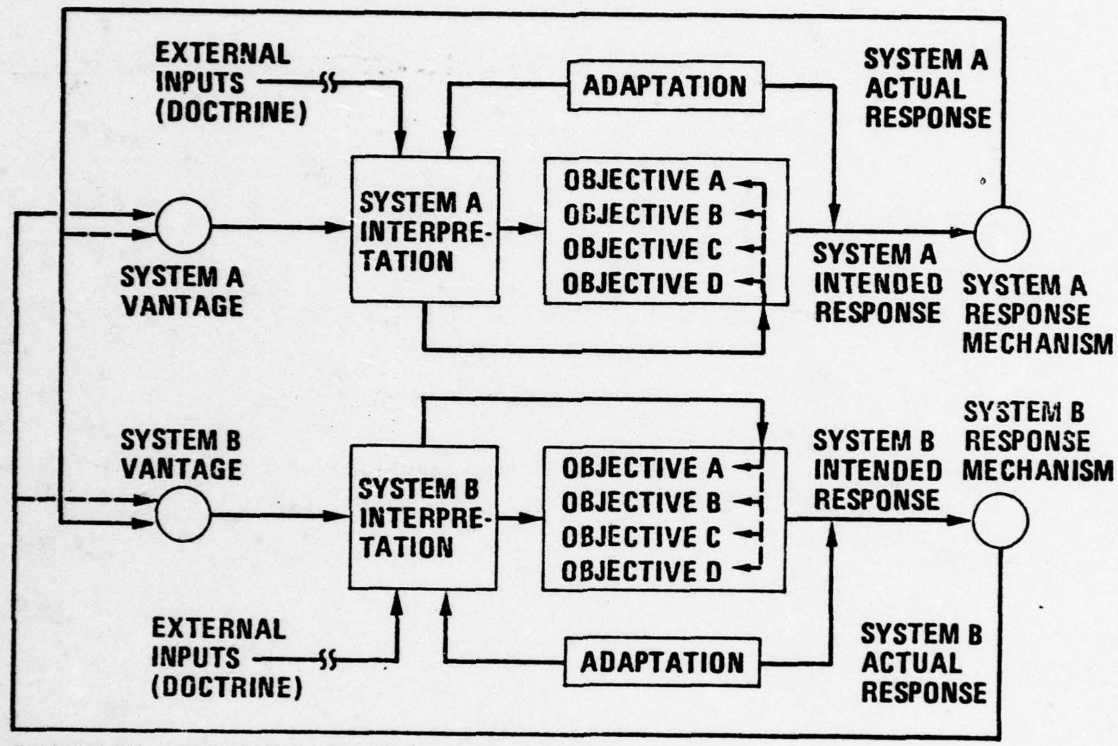
There exists, however, a class of problems for which the deterministic assumption can be demonstrated to be invalid. Such problems can generally be classed as including a variant not well defined mathematically. Such a variant might be the reaction of a human operator, or sunspot activity, or hardware failure rates. Very often, experimental data will give varying responses no matter how closely the experimental variables are controlled.

The classical approach to the solution to the problem has been two-fold. The first approach advanced by statisticians and operational analysts is a statistical formulation of the controlling variables (such as mean time between failures) and an expectation of response based on a statistical description of the problem variables, thus giving a range of expected results with or without statistical weights. Such problem formulations and their solutions are discussed in detail by Box and Jenkins, or Bryson and Ho. (See references 1 & 2.)

The second approach advanced primarily by mathematicians and control systems analysts is a cybernetic approach and sought to control the interaction by adaptive response of one of the systems to the developing situation. The problematical approach arose in the air gunnery problem where the speed of the aircraft became a significant portion of the gunnery projectile as described by Wiener (see reference 3), and it became necessary to aim the guns not at the aircraft, but at where the aircraft will be at the arrival of the projectile. In these formulations the control feedback is often a function of the perceived system response. These methods are further developed by Bellman (see reference 4).

Figure (1) shows the basic problem formulation. Two systems (A and B) are involved in an interaction situation. Each system has some vantage (often referred to as information) through which it may interpret the situation. The interpretation is then matched against certain objectives and how to achieve these objectives in a given situation (including external inputs such as doctrine). An intended response is computed (arrived at by some means), this response is attempted through some control mechanism and this feeds back to the system vantage for further interpretation. To this point we have a standard control system formulation, but two additional aspects of the real problem need be considered.

DYNAMICALLY ADAPTIVE INTERACTIVE SYSTEMS
2-SYSTEM PROBLEM



EXAMPLES: AIRCRAFT COLLISION AVOIDANCE
DESTROYER/SUBMARINE FIGHT
AIR COMBAT
CHESS

FIG. 1: TWO-SYSTEM FLOW CHART

Each system, through its interpretation of the problem may alter its objectives. Additionally, based on experience it may alter its interpretation of the situation. The last two feedback loops are the adaptive ones and make the problem both non-stationary and non-linear. As a simple example, one can use the environmental control problem. A thermostat controls both a heater and an air conditioner. The object of the interaction of system A (the environmental control system) with system B (the environment) is temperature maintenance. It is significant to the problem that this objective is attached to system A and that system B is passive (no objective or counter objective). There is neither perfect information (vantage/interpretation) or control (response mechanism). The system measures temperatures imperfectly, and it controls its apparatus with some lag (i.e., when the temp rises to that required, the furnace is hot and burning, a shutdown here wastes fuel and will probably cause a temperature overshoot).

Both types of solutions to this control problem have been successfully employed. In the statistical approach, the measurement errors are statistically modeled. Some temperature lower than desired is used to initiate the furnace and some temperature higher than desired is used to initiate the air conditioner, both cease when the desired temperature is obtained. In the cybernetic approach, the

rate of change of temperature can be used to determine the level of furnace activity (such as replacing BTU's lost). The problem may be further complicated by making the environment active with compatible or uncompatible objectives as in the case of a second unseen environmental control system with the same or different objectives.

This paper will concern itself with the more complicated of these interactions, and a strictly analysis of experimental data (as opposed to a control) view of the operational aspects of such interactions. The emphasis will be on the tactics and strategies of such interactions and a measure of the expected outcomes of such encounters.

Problems that fall into the class to be discussed include:

- The aircraft collision avoidance problem-where systems A and B are represented by airplanes and the objectives of both are to avoid collision.

- The Anti-Submarine Warfare (ASW) encounter between the Destroyer or Destroyers (system A) and the Submarine or Submarines (system B). Whose objectives in each case are: (1) survival and (2) destruction of the opposing system.

- The Air Combat Maneuvering (ACM) encounter where the systems are represented by airplane or airplanes and the objectives in each case are: (1) survival and (2) destruction of the opposing system.

- The child's game of tag, where system A (the child who is it) has the objective of placing the tag, and systems B, C, D, etc. (other children) have the objective of avoiding the tag. Note that other children in the game do not act cooperatively, but act as independent systems.

- The air-to-ground encounter where system A is an airplane or airplanes and system B is an anti-aircraft sight. The objectives in each case are: (1) survival and (2) destruction of the opposing system.

Each of these interaction problems is characterized by imperfect information and adaptive feedback mechanisms. They also embody the ability to reorder the priority and even the structure of objectives depending on the situation at hand.

The number of degrees of freedom embodied in the data limit the ability to directly analyze the vast majority of these problems due to lack of an adequate data operation

capability. The armed services, however, have recently built and begun using ranges which are fully capable of documenting the time interaction of air combat with multiple aircraft (up to 4 by 4). Such a capability provides the data needed for these analyses and, therefore, the remainder of this paper will be limited to the analysis of air combat data.

CHAPTER 2

BACKGROUND

"Experiments are the only means of knowledge at our disposal. The rest is poetry, imagination." -- *Max Planck*

Nunn and Oberle (reference 5) have highlighted the evolution of air combat. That history has been adapted with permission of the authors. It has been modified and extended for use here.

EVOLUTION OF AIR COMBAT

Air-to-air combat had its origin in World War I. It resulted from an attempt to counter the use of aircraft for tactical aerial reconnaissance. The early fighters were two-place aircraft armed with a machine gun mounted in the rear seat. The initial employment tactics consisted of flying in front of the unarmed reconnaissance plane and firing toward the rear. In an attempt to counter this tactic, a forward firing machine gun was developed to be mounted on the reconnaissance aircraft.

The first engagement between a reconnaissance aircraft with a forward firing weapon and a reconnaissance aircraft with only a rearward firing weapon took place in October of 1914 over Belgium (reference 6). A French Voisin (pusher

propeller) had been equipped with a Hotchkiss semi-automatic machine gun mounted on a pylon arrangement over the pilot's head which the observer fired. A German Aviatik was heading back toward the German lines after tactical reconnaissance. The engagement was brief but decisive. Although the two-seat German aircraft was armed with a rear-mounted gun, the French aircraft stayed low behind the Aviatik and the German plane's field of fire was blanked by its own tail and fuselage.

The German's subsequently developed a forward firing weapon which would fire through the propeller of an airplane with the engine in the nose. Shortly thereafter, allied reconnaissance aircraft were similarly armed. This technological advance not only changed the defensive capability of the reconnaissance aircraft, but actually gave birth to what is known as classical air combat maneuvering (ACM).

The introduction of the forward-firing weapon drastically altered the nature of the aerial combat. Rather than flying in front of the opponent and firing to the rear, pilots attempted to achieve a position in the opponent's rear hemisphere before firing. As the opponent was rarely cooperative, highly dynamic pursuit/evader maneuvering ensued. Aircrews very appropriately named such an engagement a "dogfight." Very early in the evolution of this dynamic air combat, referred to as ACM, fighter aircrews recognized the

need to be cautious because of the fighter's vulnerability to attack by other opposition aircraft while concentrating on a single opponent. This caution resulted in the development of various flight formations and engagement tactics designed to provide an aggressive capability without compromising the required lookout for other enemy fighters. The principles that balanced aggression and caution have remained unchanged in both character and importance to this day.

Throughout World War II and the Korean conflict, the nature of ACM remained essentially unchanged. More sophisticated gun systems and more powerful aircraft were introduced, but the basic maneuvering characteristics of the ACM engagement remained. Throughout these conflicts, aircrews adjusted engagement tactics fo fit changing tactical situations and continually relearned the well-known tactical lesson--you never see the one that defeats you.

World War II did generate a new fighter mission--the fighter interceptor. In this role, the high-speed, well-armed fighter aircraft were vectored to intercept and engage the heavy bomber as it proceeded to its target. Engagements of this type were generally characterized by high-speed fighter firing passes with some evasive maneuvering to avoid the bomber gun defenses. In the interceptor role, the forward firing weapons of the fighter were countered by

trainable weapons on the bombers. The trainable weapons were defensive weapons while the forward firing weapon was the offensive weapon.

In the mid-1950s, the air-to-air guided missile appeared in the fighter armament inventory. Although both radar-guided and heat-seeking varieties were available, the heat-seeking missile represented the technological breakthrough which most strongly affected the ACM engagement. Although the radar-guided missile was conceptually an all-aspect weapon, the difficulty in maintaining radar track on a maneuvering opponent greatly diminished the effectiveness of this missile in the classical ACM role, and the radar-guided missile was relegated to use in the interceptor role.

The heat-seeking missile, on the other hand, had the very desirable characteristic of pursuing the target after launch, independent of postlaunch maneuvers of the firing aircraft. This significantly decreased the vulnerability of the firing aircraft during the weapons-firing portion of the engagement. As with gun systems, the early heat-seeking missiles were effective only when fired in the opponent's rear hemisphere. Again, the major technological advance failed to alter the dynamic maneuvering nature of ACM.

By the early 1960s, the role of the pure fighter had essentially been replaced by the dual-mission fighter interceptor. Armed with both radar and infrared (heat-seeking)

guided missiles, the fighter interceptor was prepared for either mission. This seperation of function--radar-guided weapons for head-on, closing shots and heat-seeking missiles and guns for the ACM engagements--continued into the 1970s.

In these latter years, widespread use of the digital computer and the development of usable rapid lock-on modes of radar operation significantly increased the usefulness of radar-guided missiles in ACM engagements. Simultaneously, breakthroughs in seeker sensitivity resulted in heat-seeking missiles with some capability in an opponent's forward hemisphere. Current fighter interceptor aircraft are armed with a mix of weapons, each type useful in either kind of engagement. It should be noted that all of the current fighter aircraft have only forward firing weapons, while several of the bomber aircraft (Soviet and U.S.) have rearward firing weapons. The forward firing weapon is inherently offensive, while the rearward firing weapon is inherently defensive.

As the pure fighter was replaced by the dual-mission fighter interceptor, aircrews trained for perfection in both roles. The training engagement generally consisted of a forward quarter simulated intercept during which weapons would be employed, followed by an ACM engagement after the initial pass. During each portion of the engagement simulated missiles are launched and defensive maneuvers evaluated. In

the interceptor role, success can be equated with the ability to set-up and launch forward-hemisphere weapons, counter opponent firings, and prepare for ACM. In the ACM role, success is equated with maneuvering effectiveness and visual information, i.e., the fighter crew must maintain a clear picture of the constantly changing tactical situation, out-maneuver the opponent, obtain a position in the opponent's rear hemisphere, and fire weapons. The development of methods to evaluate the total system (aircraft, aircrew, weapon system, and tactics) in these training flights has been a goal that has eluded combat analysts for the entire history of aerial combat.

While the underlying principles have remained essentially unchanged, the enlargement of the basic fighter mission, the increase in aircrew workload, and the demands of coordination and timing have drastically altered air combat training requirements. Experience in both Southeast Asia and the Arab/Israeli conflicts, reemphasized the need for continual, comprehensive aircrew training. These training demands stimulated the development of instrumented ranges for the required routine air combat training. Developed on the principle that "seeing is believing," these ranges permit aircrews to review training engagements to isolate mistakes, evaluate weapon firings, and discuss tactical achievements and deficiencies. These ranges are described in detail later

in this chapter. The training potential of these ranges will take years to exploit. The model in this study is designed to contribute to this exploitation.

AIR COMBAT EVALUATION TECHNIQUES

The development of methodologies for evaluating air-to-air engagement has proceeded along two lines. One approach has been to analyze the maneuver dynamics of two engaging aircraft. Such models are based on physical principles and have been used to improve airframe design, drive manned simulations, and assist in postulating engagement tactics. Postulated tactics were then flight tested often with unexpected results. The second modeling approach is event oriented and consists of formulating simple measures of effectiveness of the various elements (aircraft, weapon system, tactic, etc.) of ACM engagements, and computing these measures with the data gathered at a test range.

Energy-Maneuverability Models

These models are used to analyze the maneuver dynamics and have been the most successful for ACM analysis. The basic ideas of energy maneuverability (Boyd and Christie) have been extended using variational and differential gaming techniques to determine optimal maneuver tactics. This methodology uses the physical equations of motion to relate the effects of accelerated flight on system energy (E_s) and

rate of change of energy (P_s). It has been used to design maneuver tactics based on energy management (an obscure term meaning do things the way that aerodynamics allows most easily) and to identify flight regimes where fighter airframe performance exceeds that of a postulated opponent. The methodology also helps a pilot understand the limits and potential performance of opposing aircraft. The latter is of signal importance because actual practice with opposition aircraft is seldom available. With this methodology, efficient energy management may be derived which maximizes the number of maneuver options within the limits of the airframe. This use of energy-maneuverability theory is sometimes called the maximum maneuver concept and results in the tactic of engaging enemy aircraft only at altitudes and airspeeds for which maneuver options are favorable for the aircraft under consideration.

Variations of the energy-maneuverability models have been successfully used in "man-in-the-loop," real-time ACM simulations. These simulators are adaptations of the differential equations governing motions of powered vehicles in three-dimensional space in response to actions by the pilots. Consequently, these simulations are more realistic than representations of engagements based only on analytic energy-maneuverability models. All presently operable systems are

either fixed base or only partially moving base, and limited to three aircraft (two versus one).

The energy-maneuverability models generally cannot be used to quantify either the effect of less-than-optimal maneuvering or use of weapon systems to compensate for the less-than-optimal maneuvering. These deficiencies are somewhat overcome in the man-in-the-loop simulators. In particular, the TACTICS II simulation (Spicer and Martin, reference 7) is an effort to incorporate weapons performance into the analysis.

Game-Theory Models

Attempts to use game-theory techniques for analysis of ACM models have generally been unsuccessful. Some limited success has been achieved in modeling pursuit/evader situations with differential gaming techniques (Isaacs, reference 8). To date, such models have been deterministic and highly idealized. However, these techniques appear to offer the potential of a detailed understanding of ACM models by offering a technique for evaluating some of the non-deterministic parameters in decision making. An excellent review of both pursuit/evader and differential games applications are given by Falco in reference 9. This is a comparison of three such models which compute the projected outcomes of aerial engagements. Some elements of general solutions and a

limited approach to terminal condition analysis by game theory models is given by Peng, et al in reference 10.

Test Range Models

The general analysis trend with test range models was to analyze individual engagements to see what could improve or change the outcome. The pilot it was felt, was an information gatherer and interpretor, and if he was provided with all the information (perfect information), then with the appropriate training (doctrine), the response should be known and provided by the pilot (perfect pilot).

Such a system of analysis is important to the training aspects of engagement dynamics. Feedback in the form of what information to look for, how to interpret this information, and what tactics are required to overcome performance deficiencies is valuable in bringing about the highest level of proficiency in execution. It is precisely this reason that led to the development of the test ranges.

These types of analyses, however, are meaningless in relating what a trained (proficient?) squadron of men and equipment will do against a viable opponent. The main difficulty lies in the assumptions embodied in the previous analyses:

- Perfect information does not exist. Even in an information rich environment, the pilot cannot assimilate all the information available to him.

- Perfect pilot does not exist. No matter how well trained the pilot is, he cannot respond exactly as desired (some may do better than others) even if the needed response is known.

- Doctrine is incomplete. All possible circumstances provide a continuum of events and at best only a finite subset can be considered.

These points verify those raised in the introductory section, and in fact, when several engagements are taken from the same initial conditions and executed through the same tactics, varying results are obtained (this will be demonstrated later in this paper).

In order to counter these problems arising from the non-deterministic outcomes of air combat maneuvering (ACM), three distinct analysis techniques have been developed. The technique used is dependent upon the intended use of the ACM data. The first technique stems from the desire to continuously monitor airplane performance during an ACM encounter, and follow the progress of events before, during, and after a specific occurrence (such as slat deployment). This type

of analysis is characterized by the development of a performance index which is purported to be indicative of the airplane's relative ACM performance. The advantage of such an analysis technique is the ease in which numerical data can be used to form conclusions and recommendations. The disadvantages of such a system of analysis are the inherent complexity, the difficulty in assigning a proper form to the performance index, and the required detailed knowledge of the computations in order to draw conclusions. An example of such an analysis technique is the airplane directional angle computation developed by the British, McDonnell-Douglas, and NASA Langley, for use with the Harrier(AV-8A) Vectoring in Forward Flight (VIFF) program.

The second type of analysis stems from a desire for simplicity in computation and usability of the data by people not involved in the actual computations. Such an analysis technique is represented by a discrete computation based upon a given set of conditions (such as firing opportunities). The advantages of this type of computation are its inherent simplicity, basically self-explanatory conclusions, and ease of use. The disadvantages are that it does not account for the total range of possibilities, and it may or may not be difficult to distinguish the effect of isolated occurrences in the data. Such an analysis technique is used in most tactical manuals for pilots and is typified by the use

of kill probabilities and average energy states or box scoring of kills (reference 11).

The final analysis technique combines the simplicity of the discrete analysis (and some of its disadvantages) with the increased analysis capability of the continuous analysis. The technique is characterized by taking the continuum of interairplane relationships and breaking it into discrete segments or ACM states. The airplane or hardware is then evaluated against its ability to maintain or change the ACM state of the airplane as engagement time acculumates. The ACM state analysis technique lies between the continuum approach and the discrete approach and can be driven to either extreme. By defining a very large number of states, the analysis becomes nearly continuum and carries with it the advantages and disadvantages previously noted. By taking only one state of interest (such as kills), the analysis becomes discrete and carries with it the associated advantages and disadvantages of the discrete analysis.

The ACM state analysis technique was introduced by Oberle (reference 12) and has been used on flight data, simulator data, and computer generated data (see Nunn and Oberle (reference 5)). The technique defines ACM states: offensive weapons, offensive, neutral , defensive, and defensive fatal. The analysis consists of measuring the relative ability of the engaging combatants to convert

between states by assuming that the trial engagements are a realization of either a semi-Markov or second order semi-Markov[1] process. For specific analytic details and examples, see Oberle (reference 12) and Nunn and Oberle (reference 5). Such an array of states provides simplicity of computation and easily used data (self-explanatory). Because the analysis is not continuous, it does not allow for optimization of events.

A reduction of the number of states from five to three: firing an opponent, maneuvering with opponent, and fired upon an opponent, results in a finite state semi-Markov model known as the Firing Sequence Model. The model has been used for computing the probabilities of win, loss, and draw as engagement time accumulates. For further analytic details and illustrative examples, see Nunn and Oberle (reference 5).

The analysis technique used for this research falls in the category of the continuous analysis techniques. A figure-of-merit or performance index is computed at each point in the engagement. The time variance of the figure-of-merit is then given as the engagement trend. Figure 2 gives two performance indices presently in use for reference.

---

[1] In a Markov process the transition probability between states are dependent upon current states and independent of prior states. These transitions occur without regard to time. The semi-Markov process considered is a Markov process where time in state is a random variable whose density function depends upon the state. A second order Semi-Markov process is a semi-Markov process whose transition probabilities are conditioned upon the prior as well as current state.

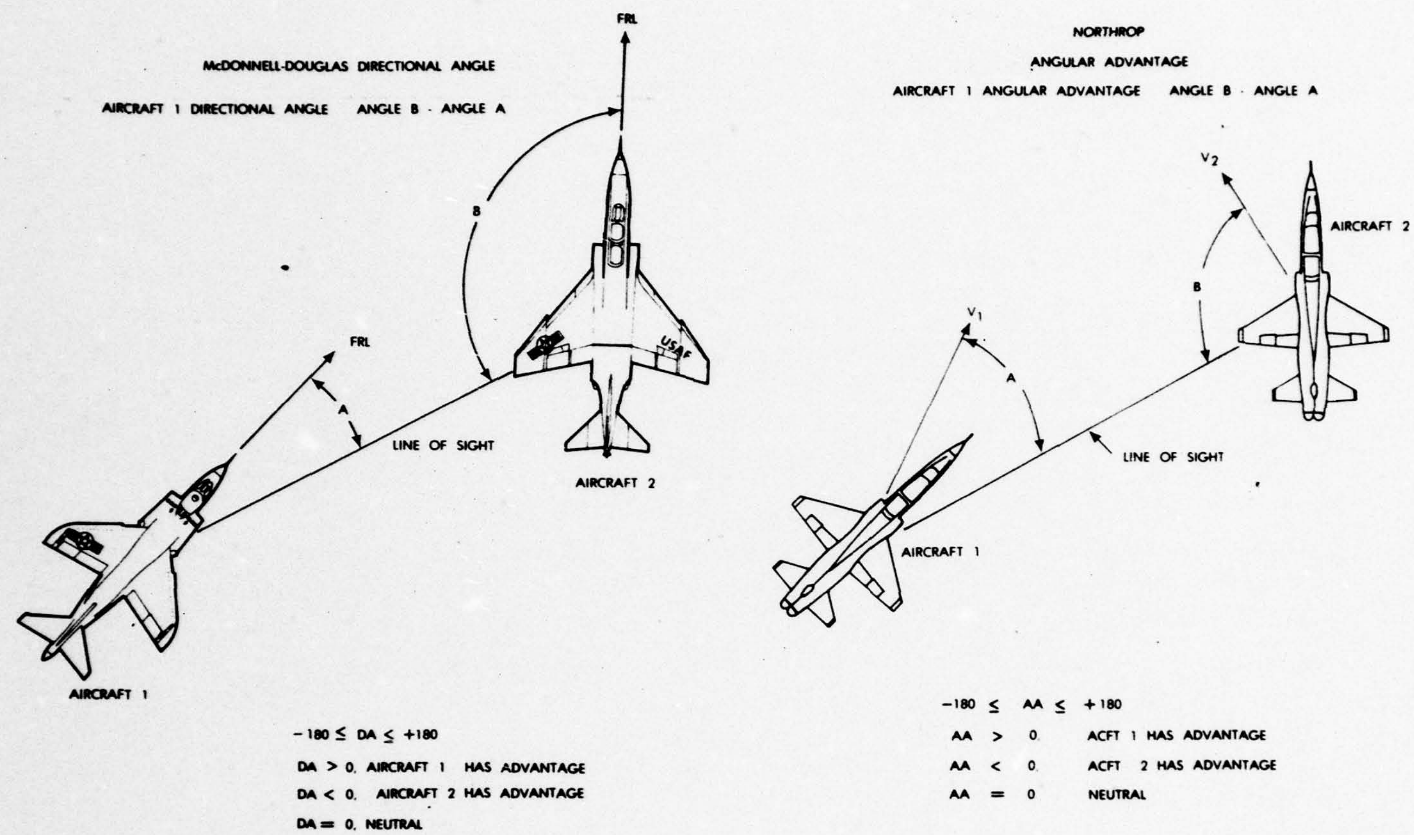


FIG. 2: EXAMPLES OF CONTINUOUS ANALYSIS PERFORMANCE INDICES

The two measures of figure 2 are tailored to turning performance, and thus include only angular terms. Northrop also uses a differential energy integral defined as:

$$I_{\Delta E_S} = \int_0^t (E_{S_1} - E_{S_2})dt \quad (1)$$

This is designed to measure the time advantage of a thrust minus drag or thrust-to-weight differential. Other indices exist, but are also tailored to a specific aircraft performance trait and are not indicative of the relative ACM position.

The second type of analysis (box scoring) is the most wide spread and well known. This is primarily because prior to 1974, air combat data even from mock engagement trials was very difficult to get. Radar tracking data, together with time correlated onboard data had to be used to construct numerical data.

Analysis of ACM testing is complicated by the two sets of data required from each aircraft. The first set of data consists of those parameters referring to the specific aircraft. These aircraft parameters are typified by airspeed, Mach number, load factor, angle of attack, and altitude. The second set of data consists of those parameters referring

to the interrelationship of two or more aircraft. The inter-aircraft parameters are typified by range, closing velocity, and relative angles.

The two indices of figure 2 reflect the primary differences in data gathering. The McDonnell-Douglas Directional Angle term requires both onboard and radar track data to compute the relative positions of the fuselage reference lines, while the Northrop Angular Advantage term requires only radar track data. The difference being the measures of angle of attack, sideslip, etc., which can only be measured by on-board instrumentation.

The availability of air combat data changed radically with the operational availability of the Air Combat Maneuvering Range (ACMR) in Yuma, Arizona, in 1974, and the Air Combat Maneuvering Instrumentation (ACMI) range at Nellis, AFB, Nevada, in 1976.

The ACMR was developed for the U.S. Navy by the Cubic Corporation for use in pilot training and research, development, and operational test and evaluation of ACM problems. The total system consists of an area of controlled airspace about 50 miles east of Yuma, Arizona, with tracking stations and radio link to a computer complex for display and communications. The system is capable of handling up to four aircraft for real-time ACM analysis. A complete description of the ACMR is contained in reference 13.

The ACMI was developed for the U.S. Air Force by the Cubic Corporation for use in pilot training and the ACEVAL (as well as other) tests. The system is basically an extension of the ACMR and is located at the Nellis Air Force Base, about 10 miles north of Las Vegas, Nevada. The system is capable of handling up to eight aircraft for real-time ACM analysis. A complete description of the ACMI is contained in reference 14.

The two systems are the same in basic concept and may be characterized as inertially aided multi-lateration systems designed to accurately track, monitor, and record high performance aircraft data. Using simultaneous range measurements from multiple ground stations, a real time multi-lateration computation uniquely determines the position of the aircraft with respect to the ground reference network. Inertial data communicated from the aircraft pod via an integral data link permits determination of aircraft attitude. These data are resolved into a situation display and alpha-numerics of the engagement particulars and are made available for display at the control center. Such resolved data are recorded on magnetic tape for later playback and debriefing. These functions are provided by the following sub-systems:

- Display and Debrieifing Subsystem (DDS) which includes three-dimensional situation displays, alpha-numeric, and status displays.

- Computation and Control Subsystems (CCS), a large multi-processor for real-time computation.

- Tracking Instrumentation Subsystem (TIS), a high-speed phase-comparison ranging system.

- Airborne Instrumentation Subsystem (AIS), a self-contained pod with sensors for measuring and transmitting airplane parameters.

Display and Debriefing Subsystem

The DDS provides real-time three-dimensional views of the flight exercise. This subsystem provides the data entry capability for the ACMR and ACMI Missions, ordnance, type of airplane, scoring criteria, and other pertinent data required for the operation are automatically entered into the system. The DDS provides a wide variety of real-time data outputs. Three-dimensional views of the range activity to include coordinate transformation as well as reorientation to the pilot's cockpit view are available upon selection. The DDS provides full debriefing capability and permits viewing scenes and operations that were not viewed during the real-time operations. Summary data of the flight operations, including scoring, are provided automatically at the conclusion of a mission. Further details are contained in reference 13.

Command and Control Subsystem

Computation of airplane position, attitude, interairplane parameters, missile trajectories, flight safety conditions, and identification is performed by the CCS. The location of each airplane is correlated with both missile and airplane characteristics for real-time display, recording on magnetic type, hard copy print out, and replay. The CCS is a mobile data processing center utilizing three SIGMA-9 computers plus normal peripherals. This data processing system provides real-time outputs pertaining to the airplane state vector, interairplane parameters, and missile simulations. The CCS provides the total state vector of multiple airplanes by using Kalman filtering techniques and incorporates over 250,000 computer instructions for real-time[2] data reduction. Further details are contained in reference 15.

Tracking Instrumentation Subsystems

The TIS consists of a master station and six unmanned remote ground stations dispersed throughout the range area (operating under the direction of the CCS). The function of the TIS is to simultaneously track and identify airborne targets and provide two-way data and voice communications between the airplane and the ground stations. Two-way data

---

[2]Real-time in this instance means within 200 milliseconds which under some conditions may be termed near-time.

communication is also provided between the CCS and the on-range communications network. The CCS makes possible read-out and correction of airplane attitude and heading references as well as monitoring of airplane weapons status, firing parameters, and other mission-dependent data. Further details are contained in reference 16.

Tracking is achieved by a high-speed phase-comparison ranging system operating on computer-controlled sequencing of the multiple ground stations. By this technique, continuous trilateration slant ranging (accurate to 4 feet) defines the position of each airplane to within 25 feet. Range measurements are taken at 100 millisecond intervals for high-performance airplanes.

Interrogation of a given airplane is commanded by the CCS through the master control station to any of the down-range units. The interrogation is uplinked to the transponder in the airplane's instrumentation subsystem. The airplane's reply is via all units in the ground station network, thereby providing multiple paths. This system redundancy is a feature that compensates for airplane shadowing and blanking of individual TIS sites.

Airborne Instrumentation Subsystem

The AIS is a self-contained pod with external dimensions and aerodynamic characteristics similar to the AIM-9 series

missile. The AIS includes an attitude reference unit, ranging transponder, data encoder, weapons bus monitor, and air data sensor. The pod is attached to the airplane in the same manner as a Sidewinder missile. The pod is wired into the airplane through existing connectors on the launch rack. Further details are contained in reference 17. Figure 3 gives a summary of the test range and subsystems.

The primary parameters to be used in the analysis are given in table 1. These data are obtained directly from the ranges. The basic interairplane parameters defined in table 1 are shown geometrically in figure 4, together with a cone of lethality.

The existence of test ranges is necessary for the type of analyses discussed, to provide both the quantity and quality of data necessary to perform the mathematical operations. Although configured primarily for training the potential of these test ranges for R&D exploitation has only just begun. This exploitation should yield significant measures of effectiveness by which air crew capabilities and tactics can be evaluated. A few such measures are delineated in this paper.

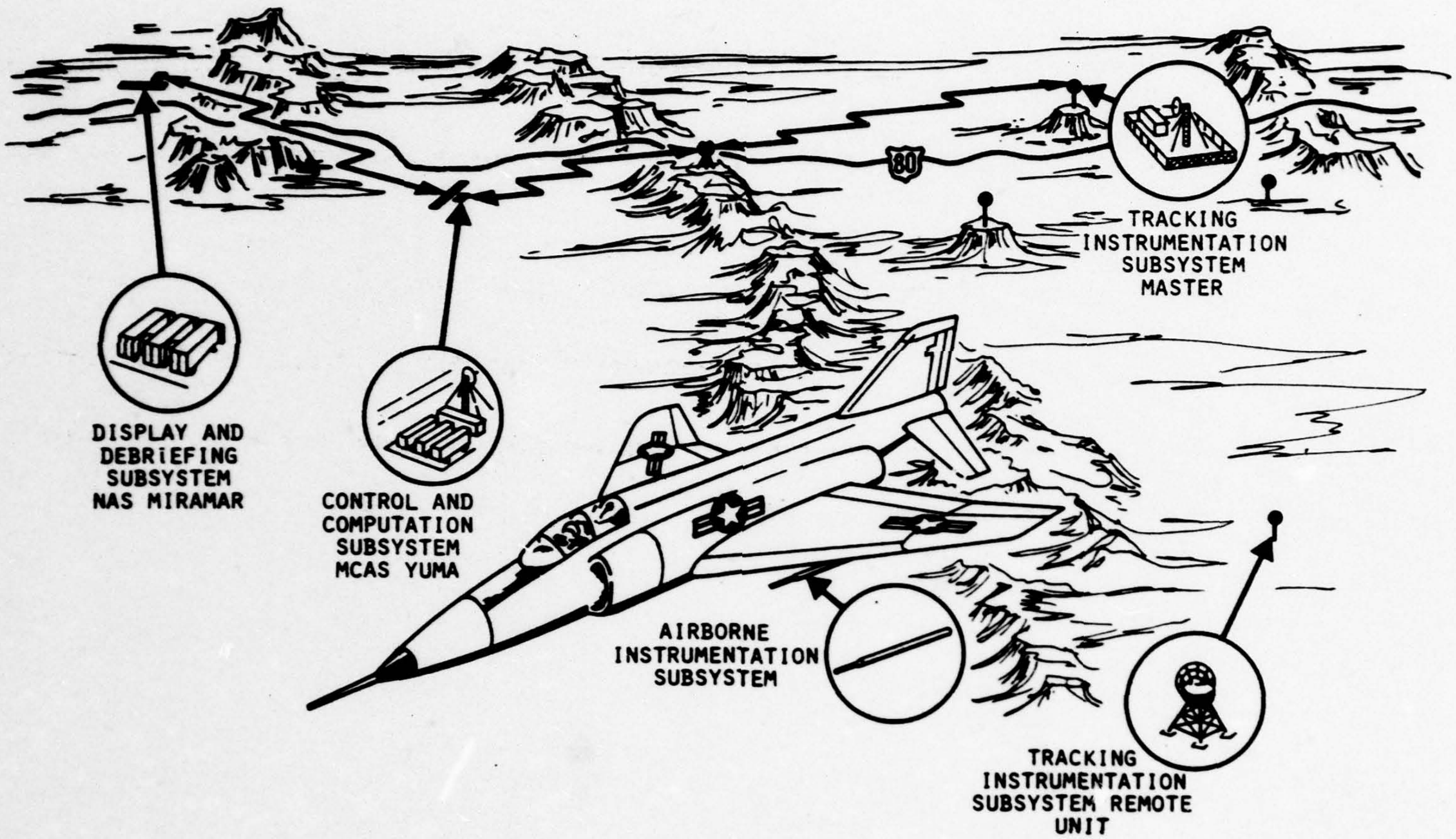


FIG. 3: TEST RANGE SUMMARY FIGURE

TABLE 1

PRIMARY ACM PARAMETERS

Airplane Parameters

Parameter	Definition
Angle of Attack (AOA)	Angle between the free stream flow and the airplane reference line
Normal Acceleration (N_z)	The load factor taken perpendicular to the flight path
Altitude (ALT)	Geometric altitude above ground level
Indicated airspeed (IAS)	Airspeed measured by AIS uncorrected for position error
Specific Energy (ES)	Sum of the weight specific kinetic and potential energies
Target Mach Number (MT)	Mach Number of the target airplane

Interairplane Parameters

Parameter	Definition
Range (R)	Line of sight distance between two airplanes
Closing Velocity (VC)	Time rate of change of range
Antenna Train Angle (ATA)	The angle between the aircraft reference line forward of the c.g. and any sight line
Angle Off Tail (AOT)	The angle between the aircraft reference line aft of the c.g. and any sight line

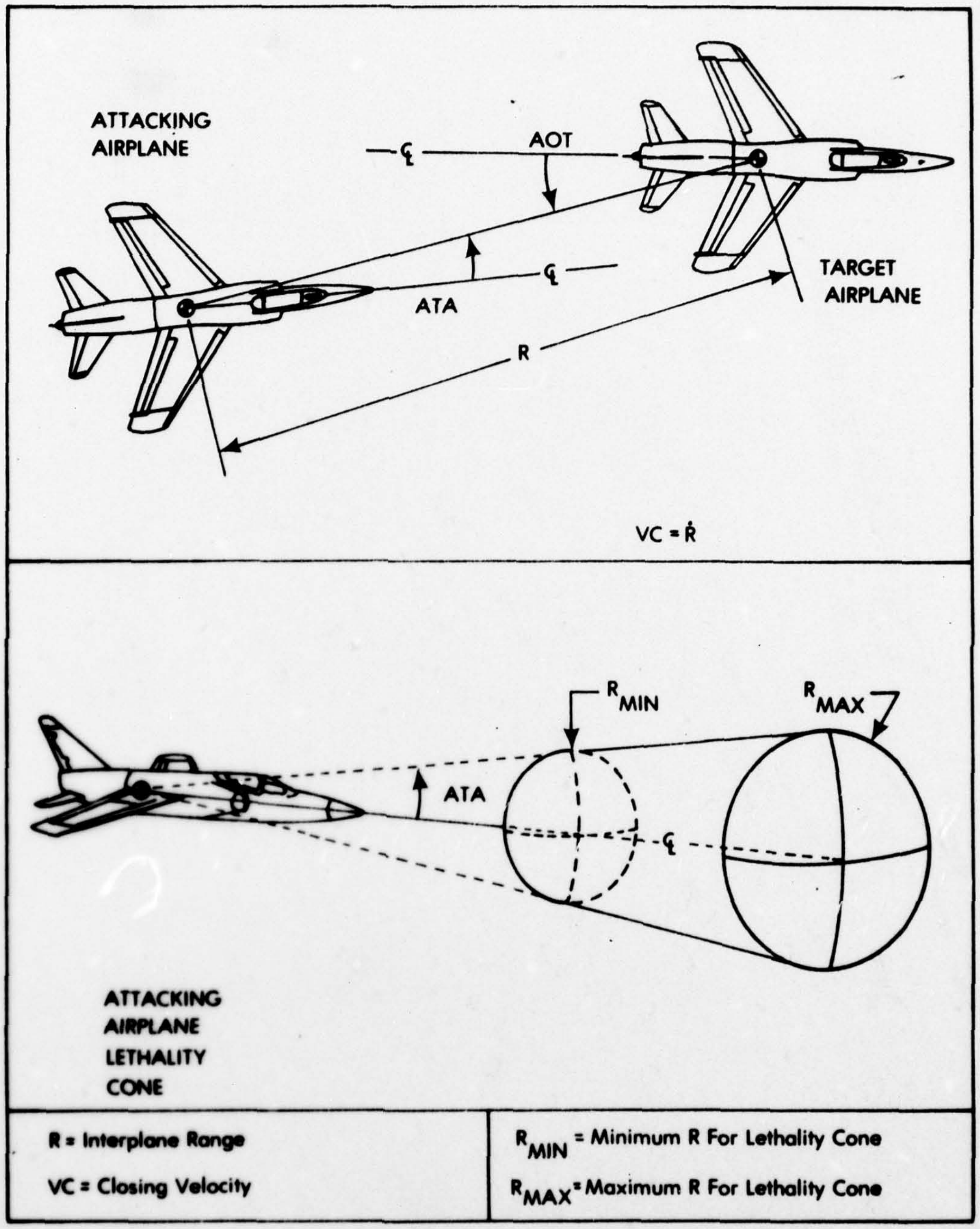


FIG. 4: INTERAIRPLANE ACM GEOMETRY

EVALUATION CRITERIA[3]

The evaluation of ACM effectiveness is sufficiently complex to warrant both analytic and flight test investigation. In fact, any ACM evaluation, whether it be of the aggressive ability of a fighter weapon system or the survivability of an attack aircraft in a hostile fighter environment, must include:

- A detailed analytic study of potential ACM ability and the formulation of preliminary engagement tactics;

- Preliminary evaluation of the postulated tactics via ACM, man in the loop, simulation; and finally

- Flight test engagements to provide the final tactics validation and effectiveness estimates.

While this sequence has been followed for the most recent ACM evaluations, the analysis of flight test data has not routinely been structurally integrated with the analysis of the first two test phases. Generally, integration has consisted of qualitative assessment of the consistency of the relative trends predicted by the first two evaluation stages with the

[3] From private communications with Dr. R.A. Oberle, reproduced here with his permission.

results of the flight testing. Additionally a few tactical measures of effectiveness are usually presented to support (not prove) the flight test assessment of ACM effectiveness. When results appear contradictory, the differences are usually treated as resulting from deficiencies in the model driving the simulation, deficiencies in the conduct of test engagements or insufficient data sample. Thus, in addition to the continued development of an analytic technique for ACM evaluation, effort is needed to integrate the analysis methodologies employed at each evaluation stage. The models presented here make it possible to numerically compare the results of "man in the loop" simulation engagements with flight test engagements. Thus, the methodologies presented in this study should be considered as complements of (rather than replacements for) the methodologies already available.

The development of a methodology for evaluating test range ACM engagements is made difficult by two factors. First, test range engagements are structurally different from wartime engagements. In wartime, ordnance is expended and vehicles are destroyed. On the other hand, during a test range engagement, weapon expenditures are only simulated. Generally, each opponent will have several weapon opportunities each of which could terminate the engagement if ordnance were actually employed. As a result, a test range engagement cannot easily be classified as a win, loss, or draw. Moreover, the large

volume of manuevering and weapon employment data available in test range engagements should lead to a methodology which can be expected to yield a deeper level of insight into the dynamics of an engagement than a simple win, lose, draw type classification. A second difficulty encountered in modeling ACM is the multiplicity of structurally different types of engagements. Starting with the basic one versus one maneuvering engagement, the complexity increases to engagements with many combatants fighting simultaneously. In addition to introducing more engaging aircraft into the scenario, it should be noted that the tactical goals of the combatants change as the complexity increases. In the less complex engagements, survivability and intelligent aggression are combined to yield very vigorous pursuit type combat. In the larger more complex engagements, survivability becomes more dominant and firing incidents occur more randomly as shots of opportunity. In this study, only methodologies appropriate to one on one and two on one engagements are developed. The evaluation of two on two and more complex engagements is discussed as an extension of the methodologies developed.

CHAPTER 3

AN INDEX OF RELATIVE WORTH

"... the crate does not matter as much as who sits in it."
-- Manfred von Richthofen
(WWI German Ace)

COMPUTING THE INDEX

The approach to the analysis of the complex problem of interaction is somewhat different than previous attempts. Because of the arithmetic growth in aircraft parameters with aircraft numbers, and the geometric growth of inter-aircraft parameters with aircraft numbers, a method of reducing the number of parameters to be analyzed is of paramount importance. This not only makes the analysis of one versus one easier to follow, but it makes the analysis of many versus many possible. In order to reduce the parameterized information, a time-variant figure-of-merit is computed.

The figure-of-merit to be used for the analysis should be indicative of the relative advantage/disadvantage of a particular set of circumstances existing in time and should not include the aircraft capability or performance which should be reflected in a time-rate-of-change of the performance index. The derivation of a performance index will be approached from an examination of the angular geometry,

interairplane distances, and interairplane dynamics. Figure 5 shows the development of the angular geometry. Figure 5 shows a definite relation between the sum of the antenna train angle and the angle off tail. This sum is also directly related to the Directional Angle (DA) term of figure 2. This DA makes a reasonable starting point for a performance index if it is normalized in order that the numerical value will have intuitive meaning.

$$DA_N = 100 \left[\frac{180 - (AOT + ATA)}{180} \right] \tag{2}$$

Equation 2 yields +100 for the best angular geometry, -100 for the worst angular geometry, and a value of 0 for the neutral condition.

The total geometry and interairplane dynamics must then modify the normalized directional angle. Figure 6 shows that the addition of the range term alters the basic conclusions. The two situations are identical in terms of angular geometry alone, but the ACM situations are radically different from a standpoint of total geometry. For example, in figure 6 the trailing aircraft is seen first in a tight trailing position which may represent an offensive weapons delivery point (guns), and second in a long trailing position which may be beyond visual sight position and therefore have no offensive value whatsoever.

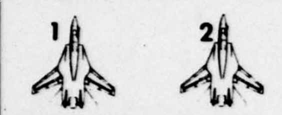
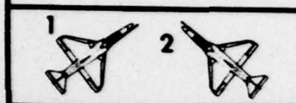
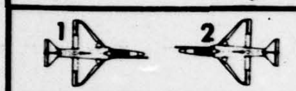
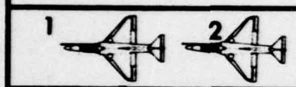
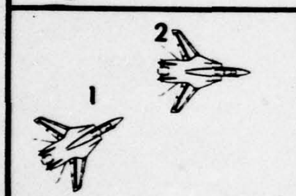
AIRCRAFT RELATIVE POSITION	ANGULAR RELATION (DEG)		QUALITATIVE ACM POSITION	
	AIRCRAFT 1-2	AIRCRAFT 2-1	AIRCRAFT 1	AIRCRAFT 2
	AOT = 90 ATA = 90 AOT + ATA = 180	AOT = 90 ATA = 90 AOT + ATA = 180	NEUTRAL	NEUTRAL
	AOT = 135 ATA = 45 AOT + ATA = 180	AOT = 135 ATA = 45 AOT + ATA = 180	NEUTRAL	NEUTRAL
	AOT = 180 ATA = 0 AOT + ATA = 180	AOT = 180 ATA = 0 AOT + ATA = 180	NEUTRAL	NEUTRAL
	AOT = 180 ATA = 180 AOT + ATA = 360	AOT = 0 ATA = 0 AOT + ATA = 0	FATAL	EXCELLENT
	AOT = 90 ATA = 0 AOT + ATA = 90	AOT = 180 ATA = 90 AOT + ATA = 270	ADVANTAGE	DISADVANTAGE
	AOT = 45 ATA = 0 AOT + ATA = 45	AOT = 180 ATA = 135 AOT + ATA = 315	ADVANTAGE	DISADVANTAGE

FIG. 5: AIRCRAFT RELATIVE POSITION CHART

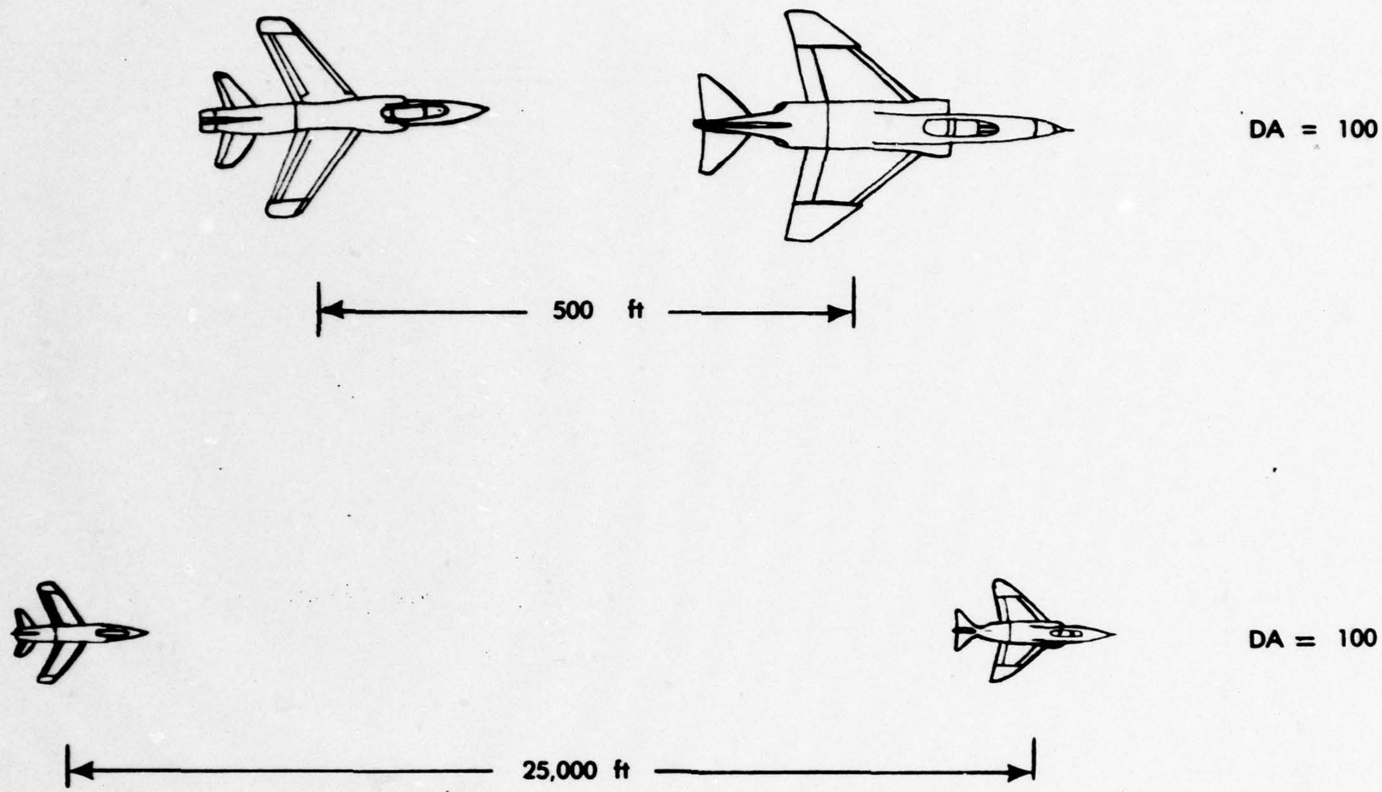


FIG. 6: INFLUENCE OF RANGE

To account for this reality, a range performance penalty function (f_r) is introduced. The f_r is shaped so that there is no penalty ($f_r = 0$) when the ranges are suitable for weapons delivery but which penalizes the performance index when the range is beyond the weapon's capability and neutralizes the fight as range becomes too large for offensive maneuvering. An optional penalty may be imposed when the offensive aircraft is at a range between a guns and a missile envelope. Such a penalty function, together with the analytic equation it represents, is given in figure 7. This penalty function is, of course, simply a curve fit of what alterations should be made to the directional angle. A total of five constants are available for controlling the shape of the curve:

- R_{MAX}--the range beyond which the engagement conditions are considered neutral.

- R_{OPT}--the range which is optimum for the delivery of an air-to-air missile.

- R_G--the range at which gun tactics begin to dominate the engagement maneuvering.

- f_{RG}--the maximum penalty for being between a guns and a missile envelope (usually taken to be small).

$$f_r = f^* \frac{R}{R_{MAX}} \left[\left(\frac{R - R_{OPT}}{R_{MAX}} \right) \left(\frac{R - R_O}{R_{MAX}} \right) e^{-8\left(\frac{R - R_G}{R_{OPT}}\right)^2} \right]^2 + \left[\frac{1}{1 + 500\, e^{-\left(\frac{R - R_{OPT}}{R_{MAX}}\right)^{12}}} \right]$$

$$f^* = \left[\frac{f_{RG} (R_{MAX})^5}{(R_G - R_{OPT})^2 (R_G - R_O)^2 R_G} \right]$$

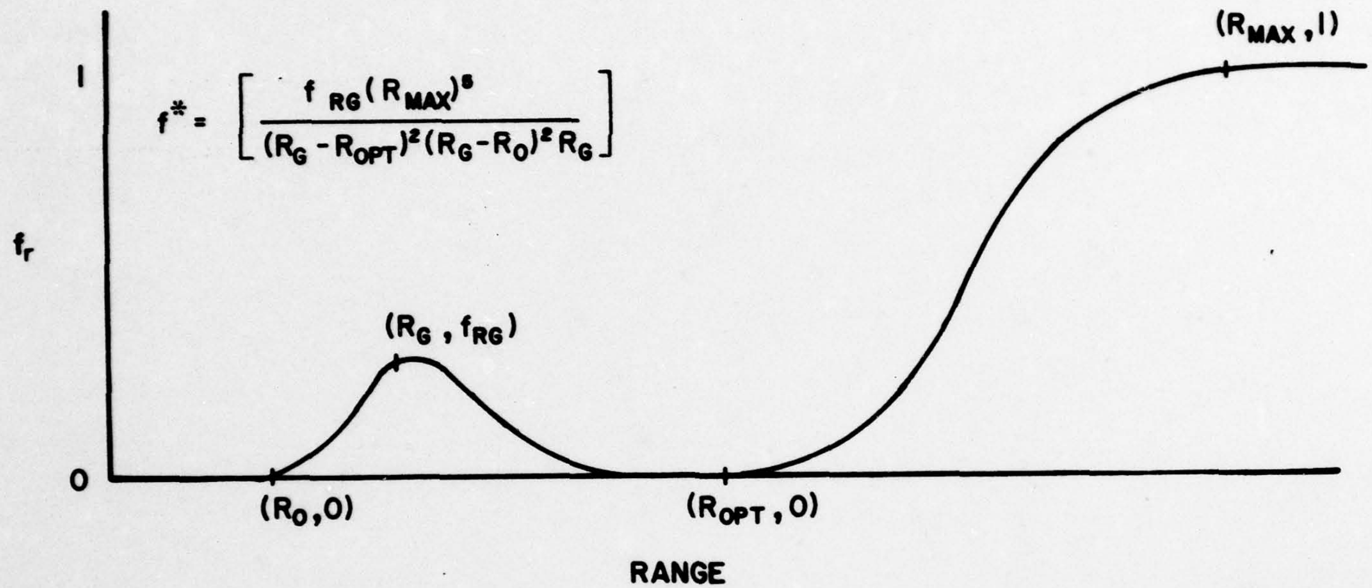


FIG. 7: RANGE PERFORMANCE PENALTY FUNCTION

- R_O--the range below which no penalty will be imposed.

R represents the interaircraft range.

The range penalty function has the general form of a sigmoid curve and would modify the DA to yield a figure-of-merit (FM) as follows:

$$FM = DA_N (1 - f_r) \tag{3}$$

The figure-of-merit is further affected by interairplane dynamics. The effect of closing velocity is illustrated in figure 8. That the two situations of figure 8 are different is not questionable, but how they are different is another matter. Just what an advantage or disadvantage is in this instance is a complicated function of range and offensive/defensive position. Certain statements can be made for a positive or negative energy increment as tabulated in table 2.

Table 2 was generated by lengthly discussions with pilots and analysts in an attempt to identify where and under what circumstances energy can be converted to position in an engagement. These complicated relations can be somewhat simplified by noting that for everything else equal, the effect of a positive energy increment is opposite the effect of a negative energy increment, and the effect in an offensive

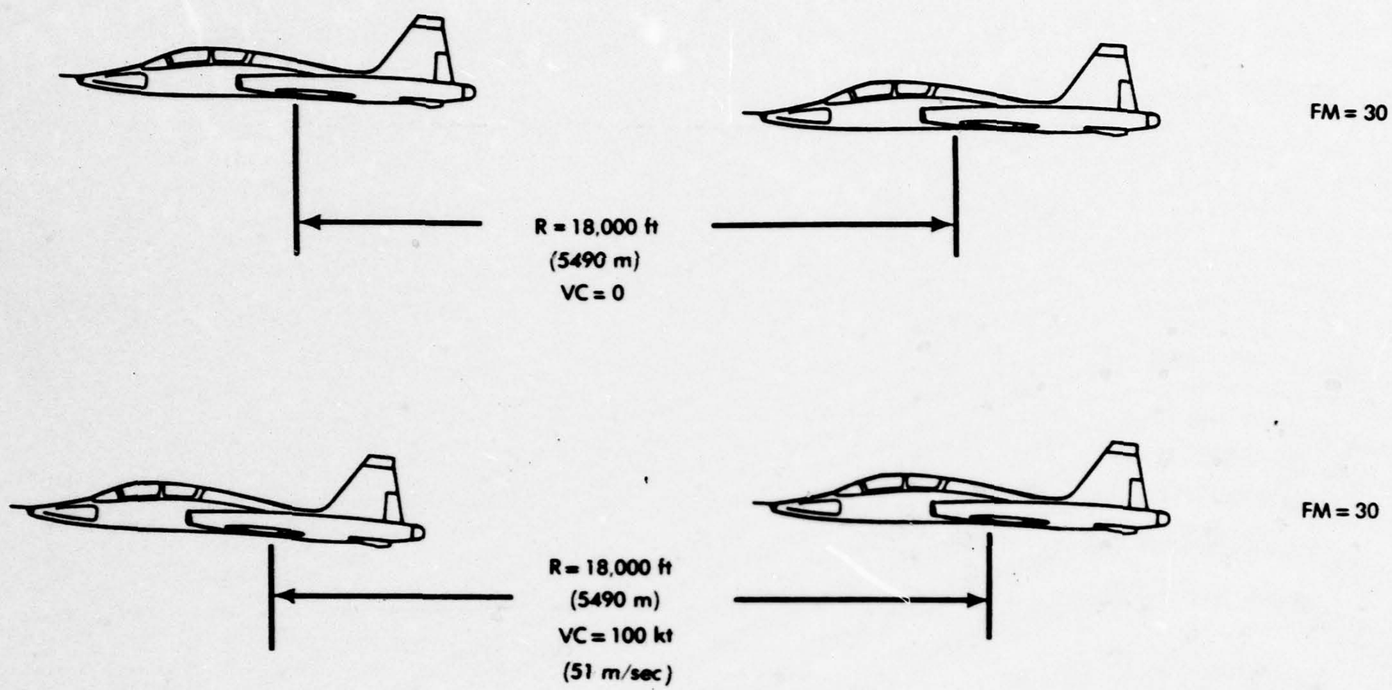


FIG. 8: INFLUENCE OF CLOSING VELOCITY

TABLE 2

ENERGY INFLUENCE ON PERFORMANCE INDEX

Range	Energy Increment	ACM State	Conclusion
Very Close	Positive	Offensive	Disadvantage due to move toward over-shoot
		Neutral	No effect
		Defensive	Advantage due to move toward over-shoot
	Negative	Offensive	Advantage due to increase of range
		Neutral	No effect
		Defensive	Disadvantage due to increase of range
Weapon Opportunity	Positive or Negative	Offensive	No effect unless weapon parameters are affected
		Neutral	No effect
		Defensive	No effect unless weapon parameters are affected
Larger than weapon opportunity but less than very far	Positive	Offensive	Advantage due to move toward weapon opportunity
		Neutral	No effect
		Defensive	Disadvantage due to move toward weapon opportunity
	Negative	Offensive	Disadvantage due to move away from weapon opportunity
		Neutral	No effect
		Defensive	Advantage due to move away from weapon opportunity
Very far	Positive or Negative	All	No effect

state is the opposite of the effect in a defensive state. A function (K) reflecting these influences, as given in figure 9, modifies the figure-of-merit and yields performance index as follows:

$$PI = DA_N (1 - f_r)K \tag{4}$$

As in the range penalty case, the energy function is simply a curve fit of what is represented in table 2. The range values (R_{MAX}, R_{OPT}, R_O, R) are as they were defined under the range function. E_S is the weight specific energy of the aircraft defined in equation (5).

$$E_S = h + \frac{v^2}{2g} \tag{5}$$

where h is the aircraft altitude, v is the aircraft velocity, and g is the acceleration of gravity. ΔE_S is the energy difference between fighter and adversary and $\bar{E}_S$ is the average energy. The quantity $\Delta E_S/\bar{E}_S$ then represents the relative percentage discrepancy in energy. Finally, the coefficient E_{dev} is available for tempering the amount of influence the energy coefficient has on the performance index.

OFFENSIVE:

$$K_{OFF} = 1 + \left[\frac{1}{1 + E_{dev} e^{-6.91 \frac{R}{R_0}}} + E_{dev} e^{-4\left(\frac{2R - R_{MAX} - R_{OPT}}{R_{MAX} - R_{OPT}}\right)^2} - 1 \right] \frac{\Delta E_S}{\overline{E}_S}$$

DEFENSIVE:

$$K_{DEF} = \frac{1}{K_{OFF}}$$

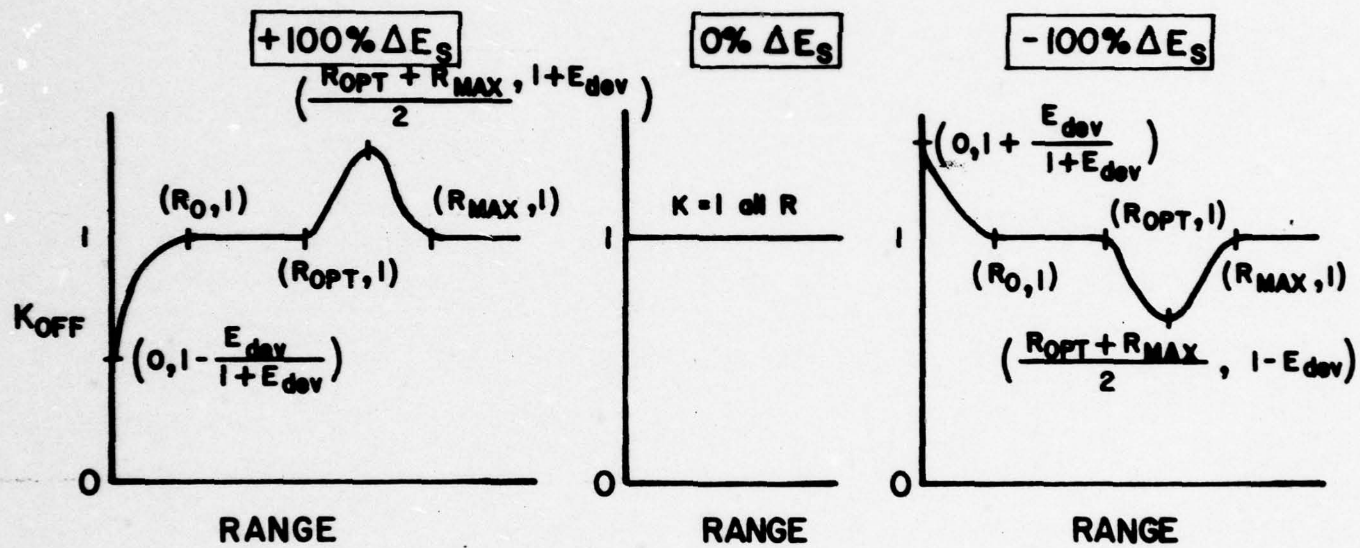


FIG. 9: ENERGY INFLUENCE FUNCTION

PERFORMANCE INDEX COMPARATIVE ANALYSIS

Maneuver Conversion Model

The maneuver conversion model is an ACM state type analysis and is the most fully developed of the non-deterministic analysis methods (Oberle, reference 12). The maneuver conversion model characterizes ACM engagements as a realization of a semi-Markov process with state conversion probabilities and time in state distributions. It was also, the first non-deterministic model to be applied to actual flight data. See Chapter 2 for further discussion.

Figure 10 is a comparison between the computed performance index (equation 4) and the maneuver conversion model (reference 12), for an actual test engagement done at the ACMR, Yuma. The figure shows close agreement in engagement trend, but the performance index allows the analyst to determine the transition points much more accurately. For example, in figure 10, the point at which the fighter begins to show a negative engagement trend is around 59 seconds, and if the defensive state is established at PI = -30, the fighter became defensive at 72 seconds into the engagement. The maneuver conversion model only indicates the point at which the fighter became defensive; figure 10 also illustrates the sensitivity of the discreet model to the accuracy of parameters. A small angular change

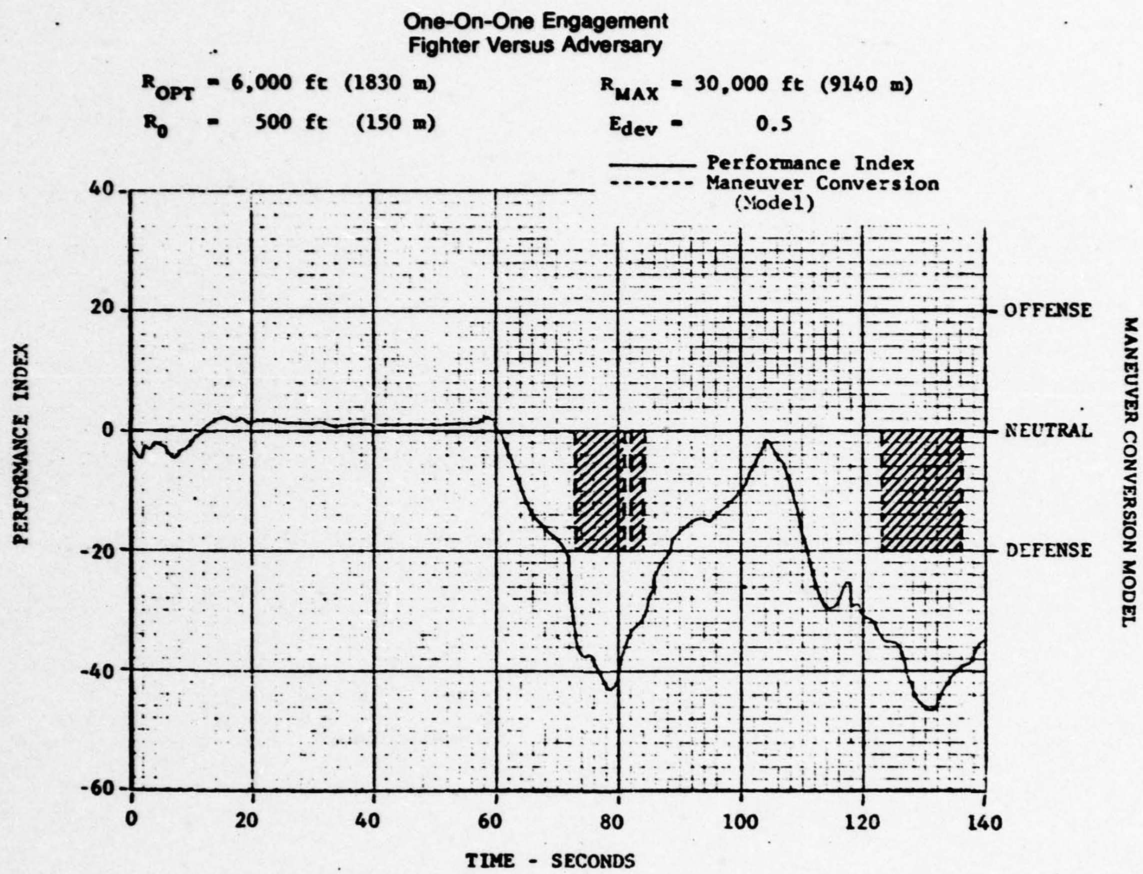


FIG. 10: COMPARISON OF THE PERFORMANCE INDEX WITH THE MANEUVER CONVERSION MODEL

(as little as 1 degree) can cause the state change that occurs at 81 seconds. No such influence is present in the performance index which has a small sensitivity to small changes in parameters and a large sensitivity to large changes in parameters.

Directional Angle

The directional angle was developed as an engagement by engagement measure-of-effectiveness in time by McDonnell-Douglas for use with the analysis of Harrier (AV-8A) air combat trials. It is primarily a measure of turning performance advantage/disadvantage and was developed to demonstrate the effectiveness of thrust vector control in maneuvering.

Figure 11 shows with the same engagement the comparison between the computed performance index and the computed directional angle with the exception that the data were normalized (to the range of +100 to -100) for purposes of comparison. The two analysis techniques give a marked difference in engagement trend during the first 60 seconds of the encounter. During this portion of the engagement, the fighter is trading angles for range (up to 36 seconds), and the directional angle analysis method does not consider this. The actual separation became 24,000-30,000 feet (7 300-9 100 m) which is enough range to suppress the performance index to neutral with the constants used. The small differences that

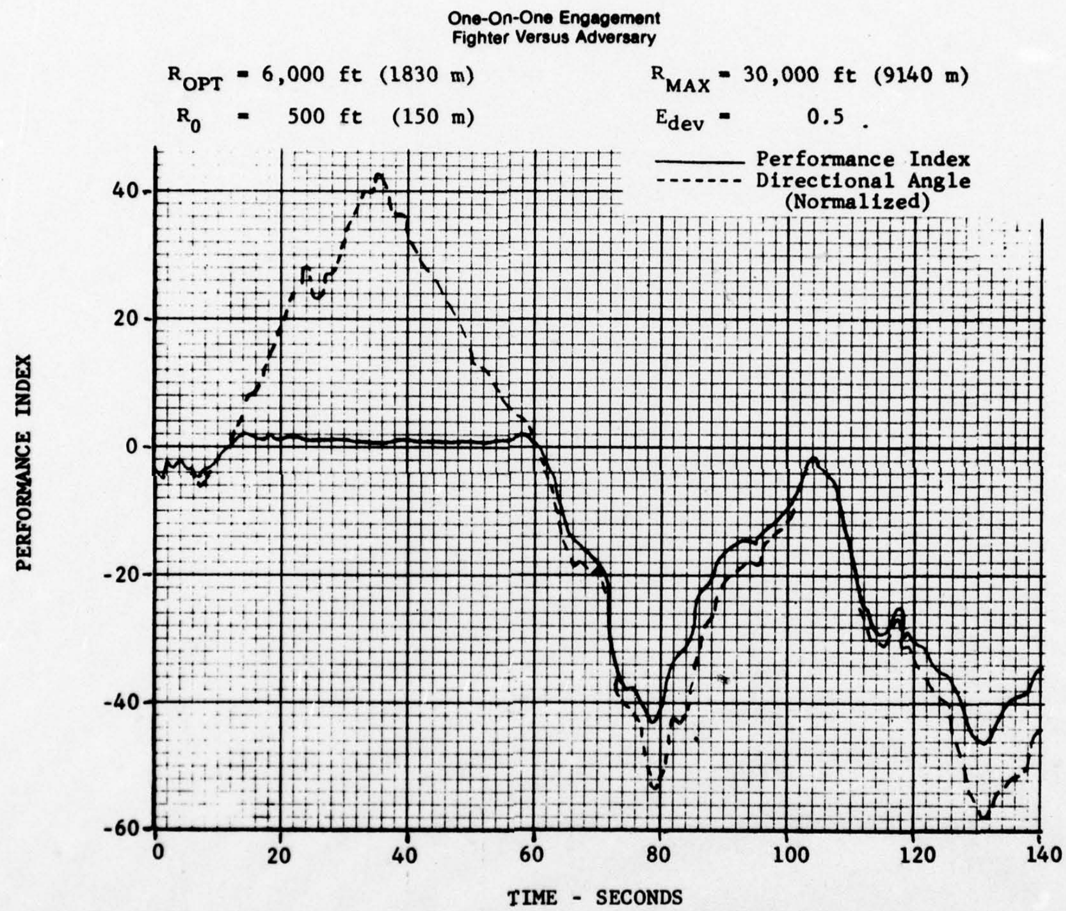


FIG. 11: COMPARISON OF THE PERFORMANCE INDEX WITH THE DIRECTIONAL ANGLE CRITERIA

occur after 60 seconds are primarily due to the energy influence which favors the fighter and reduces his defensive posture slightly.

PERFORMANCE INDEX PARAMETER STUDY

Range Effects

Figure 12 shows the effect of the maximum range parameter by computing the performance index (all else being equal) for two different values of R_{MAX}. As expected, the smaller value of R_{MAX} suppresses the engagement to neutral with the most profound effect coming between 120 and 140 seconds (this is caused by ranges between 12,000 and 16,000 feet (3600 and 4900 m)). In each of the parameters examined, the same engagement is used for comparative purposes.

In general, the R_{MAX} parameter represents the maximum interairplane range beyond which the engagement is considered neutral. This is generally a sight type of parameter but could be interpreted to be seeker head or radar ranges for individual weapons analyses. The R_{MAX} values in table 3 are based upon using sight as the engagement criteria. These data are estimated from discussions with pilots at the Naval Fighter Weapons School, VF-111, VX-4, and others. In any given application, these values may change or may even become a function of engagement parameters such as altitude, meteorology, and background (sky, terrain, sun, etc).

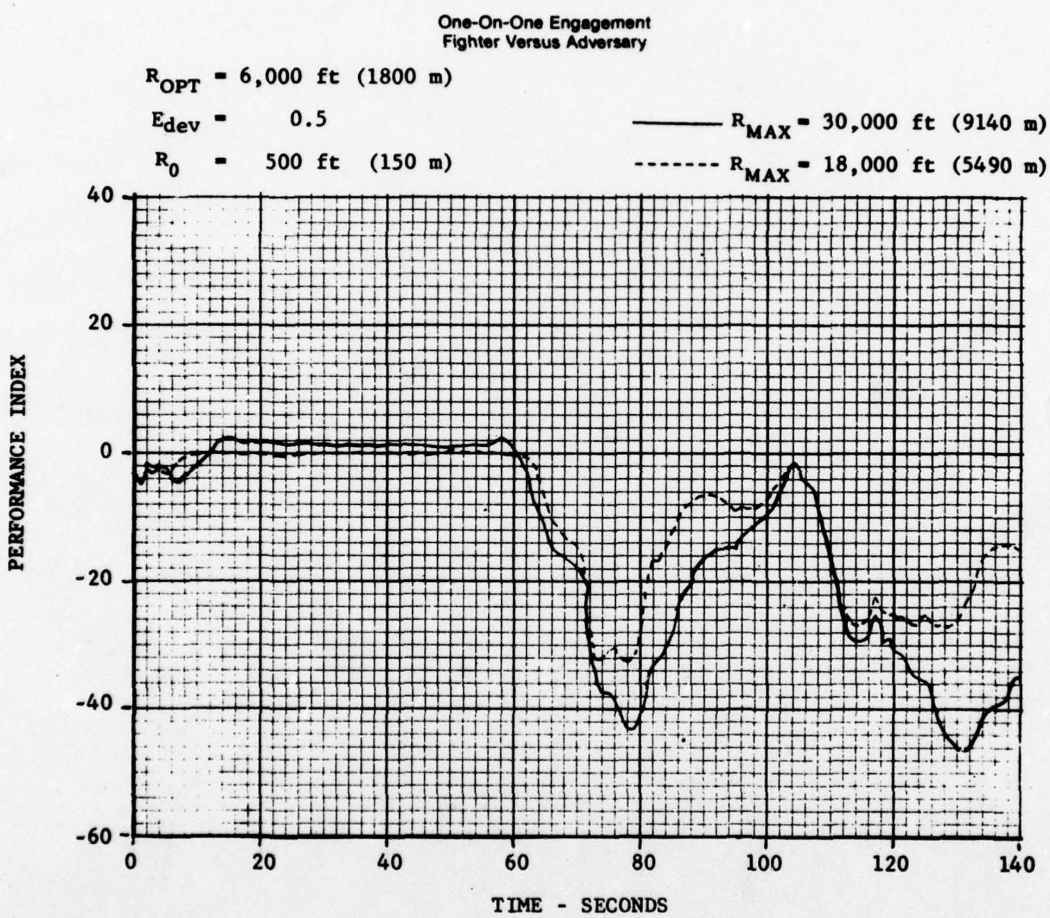


FIG. 12: INFLUENCE OF MAXIMUM RANGE PARAMETER

TABLE 3

ESTIMATED VALUE OF R_{MAX} FOR SPECIFIC AIRCRAFT TYPES

Aircraft	R_{MAX} (1)
MIG-17 FRESCO	24,000 ft (5 490 m)
MIG-19 FARMER	26,000 ft (7 920 m)
MIG-21 FISHBED	27,000 ft (8 230 m)
MIG-23 FLOGGER	27,000 ft (8 230 m)
MIG-25 FOXBAT	30,000 ft (9 140 m)
F-4 PHANTOM	30,000 ft (9 140 m)
F-8 CRUSADER	24,000 ft (7 320 m)
F-14 TOMCAT	27,000 ft (8 230 m)
A-4 SKYHAWK	24,000 ft (7 320 m)
F-5 TIGER	18,000 ft (5 490 m)
F-106 DART	30,000 ft (9 140 m)
F-15 EAGLE	27,000 ft (8 230 m)
F-18 HORNET	26,000 ft (7 920 m)

NOTE: (1) Select R_{MAX} corresponding to defensive aircraft.

Appendix B gives data pertinent to ACM analyses for the aircraft as taken from references 19-21.

Figure 13 shows the influence of two different values of optimum range. The expected result of minimum influence for moderate change is desirable because the curve should generally be flat in the area of an optimum missile launch. This also allows some latitude in the choice of the optimum range parameter. The optimum range parameters chosen for this comparison are not representative of a typical weapon, but do fall within the values generally associated with the AIM-9 or ATOLL heat seeking series of missiles, currently employed in large numbers in many countries (including communist block countries).

Energy Effects

Figure 14 shows the computed value for two different values of energy coefficient (E_{dev}). The effects of this parameter are subtle with the fighter enjoying about a 33% advantage in specific energy. A long-term energy advantage will be reflected in the index as a range or angle influence. The effect will be much more pronounced in a slashing type of engagement (high energy) where the fighter may enjoy as much as 100% energy advantage. The coefficient E_{dev} may be chosen to yield the energy effect desired. Test cases used 0.5 for computation. In general, E_{dev} will be between

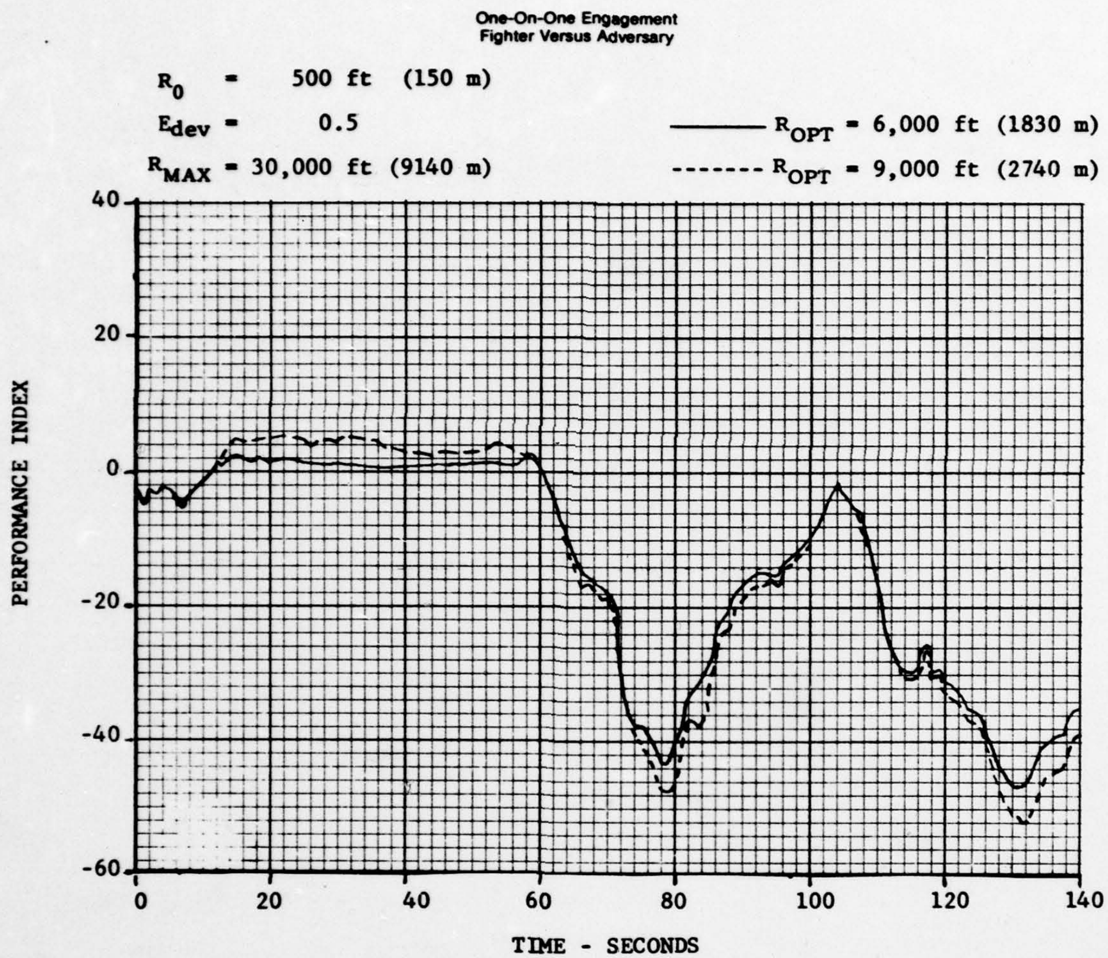


FIG. 13: INFLUENCE OF OPTIMUM RANGE PARAMETER

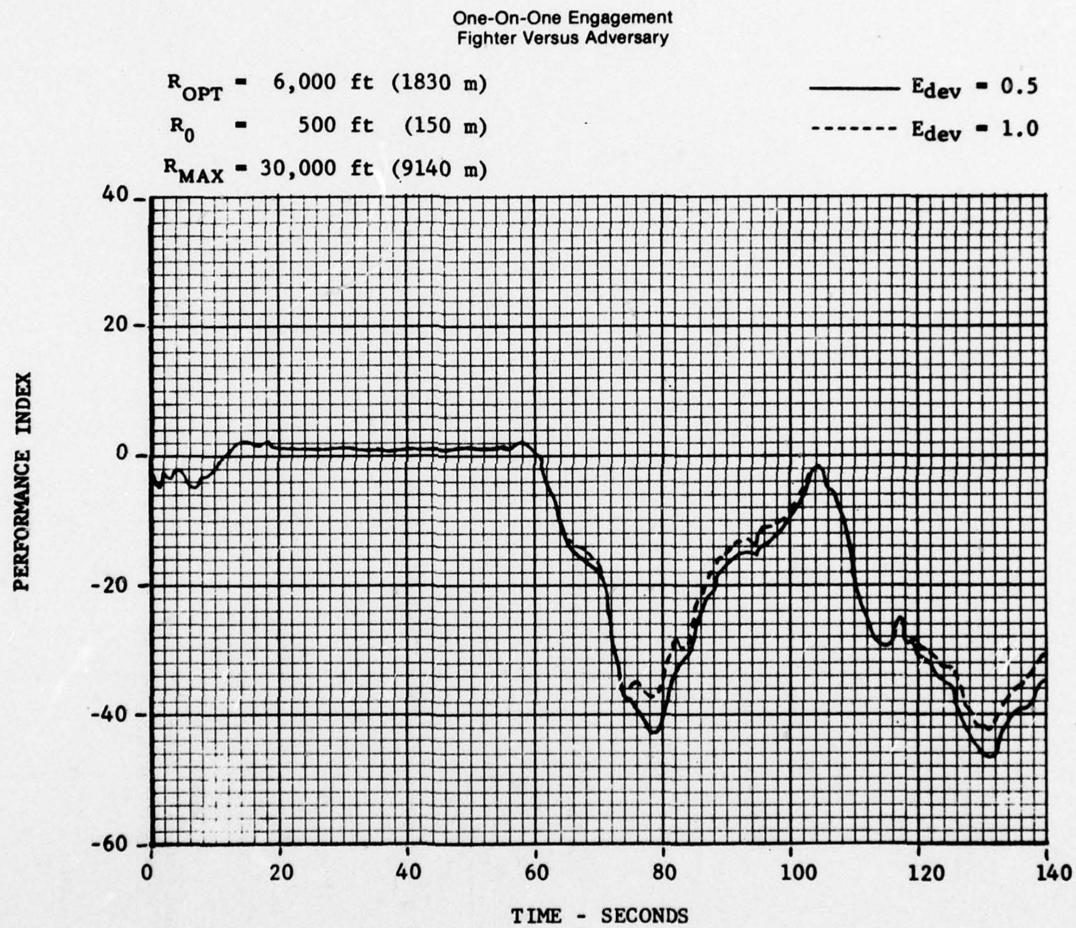


FIG. 14: INFLUENCE OF ENERGY COEFFICIENT

0 and 2.0 and is chosen to yield the desired magnitude of energy influence.

Interenvelope Gun Penalty

The interenvelope gun penalty was structured into the model to include the case of an aircraft carrying long-range missiles and guns with no intermediate range missile. In cases where the gun and missile envelope overlap, the penalty would be zero. Test cases have used an arbitrary .025 value for f_{RG}. Since no data sets were used where this parameter had a profound effect, sensitivity was not explored.

EXTENSION TO MULTIAIRCRAFT

One of the most difficult areas of ACM analysis is the extension to the multiaircraft situations. Each additional aircraft adds a multiplicity of complications to the problem both conceptually and mathematically. Few models attempt extension to this area even though an actual engagement has a much higher probability of being multiaircraft than one-on-one. Table 4 is an extract of reference 12 and gives the logic in constructing a two-on-one maneuver conversion model. The table was constructed through extensive analysis of two-on-one engagements and pilot interviews and is both logical and intuitive. It does not, however, follow precise mathematical trends. For example, the simultaneous existence of an offensive fighter and defensive fighter does not give a

TABLE 4

RULES FOR STATE EVALUATION OF A TWO-ON-ONE ENGAGEMENT

1. The section* is OFFENSIVE WEAPON when at least one member is in offensive weapon state and the other is higher than a fatal defensive state.

2. The section is OFFENSIVE when at least one member has an offensive position and the other is higher than a fatal defensive state.

3. The section is NEUTRAL when both members are in neutral state.

4. The section is DEFENSIVE when at least one member is in defensive state and the other is either neutral or defensive.

5. The section is FATAL DEFENSIVE when at least one member is in fatal defensive state and the other has less than offensive weapon state.

6. The section is in a TRADEOFF state when one member of the section is in offensive weapon state and the other is in a fatal defensive state.

*A section is defined as the total number of friendly or adversary aircraft involved in the engagement.

neutral section (condition 2 of table 4). The extension of the performance index will be in the same manner as the maneuver conversion model: i.e., a section performance index.

Figure 15 shows a two-on-one engagement, together with its maneuver conversion model, for both fighter-to-target pairs. This particular engagement is of specific interest because of the tradeoff situation between 50 and 60 seconds and the reversals of the ACM state present for the section. The individual indices follow well the engagement trends of the maneuver conversion model, but the combination must also follow for a section coefficient.

After many trials, a mathematical form of section coefficient was derived which could follow the maneuver conversion extension as shown in figure 16. The specific calculational procedure termed magnitude sum is given by:

$$K^* = \frac{(PI_1)\ |PI_1| + (PI_2)\ |PI_2|}{\left|(PI_1)\ |PI_1| + (PI_2)\ |PI_2|\right|} = \pm 1 \tag{6}$$

$$PI_{SECTION} = K^* \sqrt{\left|\ (PI_1)\ |PI_1| + (PI_2)|PI_2|\right|} \tag{7}$$

The absolute value accounts for the sign of the individual engagement indices. Tradeoff was established by looking at the individual aircraft performance indices and appears to lag in time but this is not deemed significant because of the sensitivity of the maneuver conversion model to small

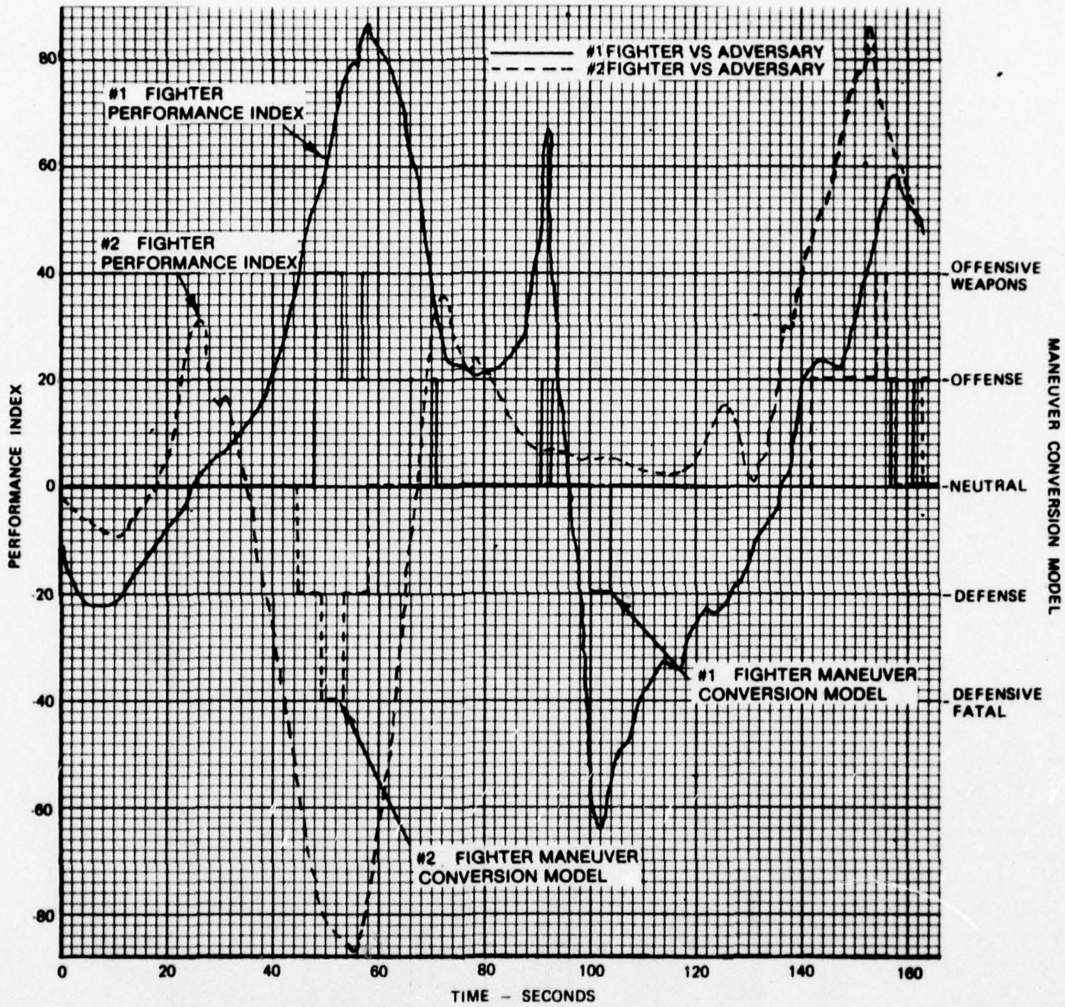


FIG. 15: INDIVIDUAL FIGHTER AIRPLANE PERFORMANCE IN A TWO-ON-ONE ENGAGEMENT

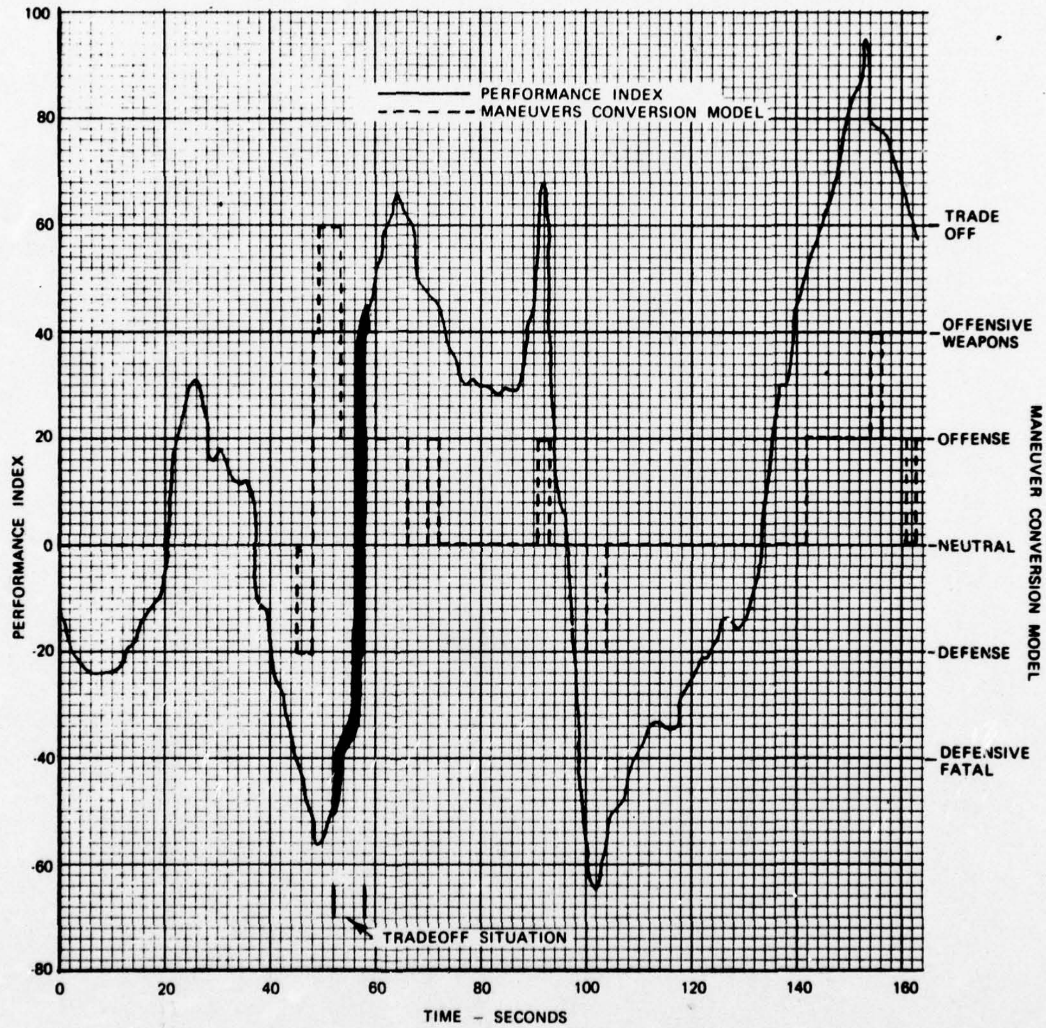


FIG. 16: FIGHTER SECTION PERFORMANCE INDEX FROM MAGNITUDE SUM METHOD

parameter changes as discussed previously. The section performance index is directly extendable to larger engagements (more aircraft) by:

$$K^* = C \frac{\sum_{i=1}^{n} (PI_i) \mid PI_i \mid}{\left| \sum_{i=1}^{n} (PI_i) \mid PI_i \mid \right|} = \pm C \tag{8}$$

$$PI_S = K^* \sqrt{\left| \sum_{i=1}^{n} (PI_i) \mid PI_i \mid \right|} \tag{9}$$

where C is a proportionally constant to establish maximum and minimum values. For $PI_S = \pm 100$, C is equal to the square root of the inverse of the number of pair possibilities.

The performance index is reflective of the trend of individual engagements as verified by comparison with other analytic techniques and critiques of air crews involved in air combat trials. As stated earlier, however, each engagement is unique and does not, in itself represent the capabilities of either aircraft or aircrews. The remainder of this report will deal with the analysis of several trials as a group, as well as individual engagement trials.

CHAPTER 4

THE CHARACTERIZATION OF MULTIPLE EXPERIMENTAL TRIALS

"In speaking about the past war, the first thing we encountered in engagement with the enemy were tens and hundreds of unexpected things."
-- Lt. Gen. of Aviation G. Pavlov, USSR

MULTIPLE TRIALS

Because in the case of dynamically interactive systems, the response to a given situation is governed by anticipation, interpretation, expectation, and other intangibles, each trial is unique. In fact, when several trials are conducted from precisely the same initial conditions, a wide variety of outcomes results. Such a variety of outcomes at first perplexed analysts who turned to statistical descriptors such as box scoring described in Chapter 2. In fact, the training of fighter pilots included a large discussion on what to do in given situations. The execution, however, often went something like -- "After two turns I could see that he would .... So instead of .... I .... -ed." While initially frustrating to the analyst it can be seen that the interaction described is highly desireable, and the adaptive feedback loops working to counter the opponent are the very ones that make each trial a unique event. The uniqueness of events is best demonstrated

in figure 17. Figure 17 represents thirty-three (33) engagements of two fighters versus one adversary as taken from data flown on the ACMR at Yuma, Arizona. The tape of similar engagements was created by the control structure shown in figure 18, and with the program logic shown in figure 19. This data extraction procedure is part of a large scale data extraction and analysis procedure known as the "ACMR Readiness Estimation System" that is now operational at the ACMR in Yuma, Az. Details of the ACMR Readiness Estimation System are described in Oberle, Naron, and Simpson (reference 22).

Figure 18 shows the breakdown of the computational methods by seven inter-active program modules as follows:

- DEBRIEF--reads data not automatically recorded by the system such as pilot comments, first visual sighting by an aircrew (tally), calls for break-off of the engagement (BUGOUT) and low fuel states (BINGO), etc.

- GETOBS--Gets the observed or recorded data.

- ACMRDB--the driver program which controls the flow of data and interfaces directly with DEBRIEF, GETOBS, and CRTREC as shown in figure 19.

- CRTREC--computes real time records by control of three calculational routines; IAP, ENVIND, SAP.

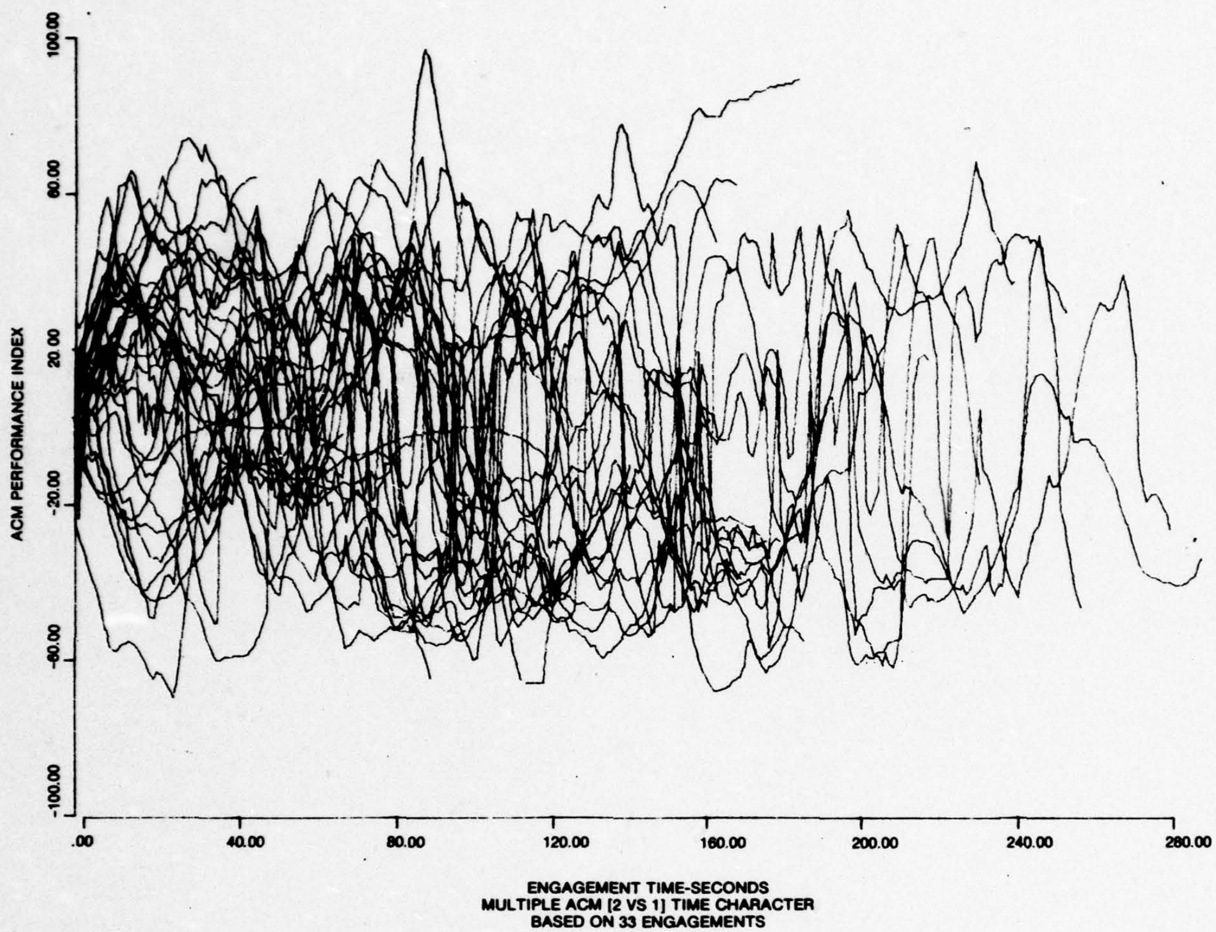


FIG. 17: PERFORMANCE INDEX TIME DATA FOR 33 ENGAGEMENTS

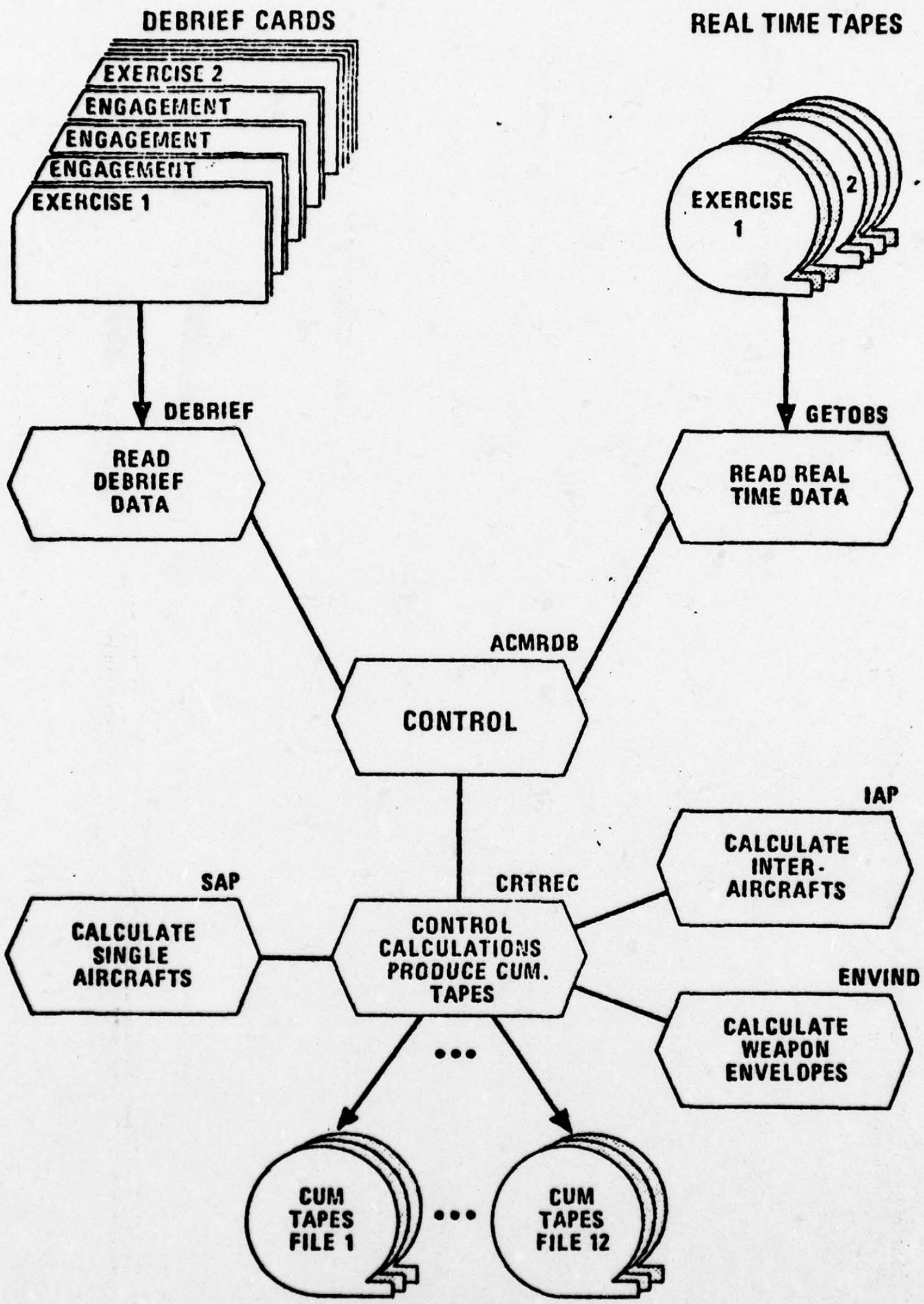


FIG. 18: CONTROL STRUCTURE FOR ACMR CUMULATIVE PACKAGE

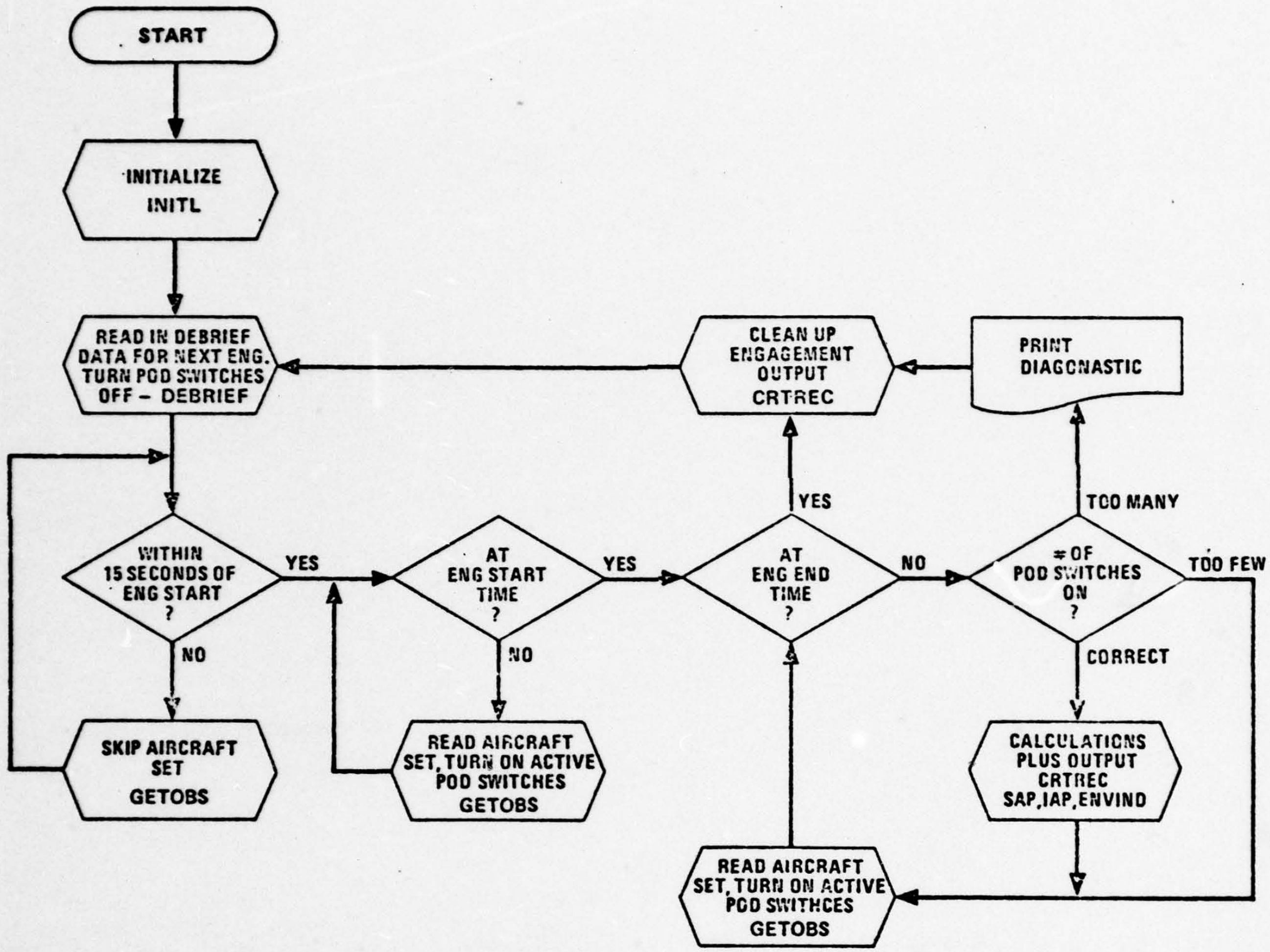


FIG. 19: PROGRAM LOGIC FOR ACCUMULATION OF ENGAGEMENTS

- IAP--computes inter-aircraft parameters for all pair possibilities.

- SAP--computes single-aircraft parameters for each aircraft.

- ENVIND--computes individual weapons envelopes to see if a firing solution has been satisfied.

The results are then output to tape for further analyses. The individual engagement tapes are then released back to the system for reuse on further engagements.

The engagements of figure 17 are fairly typical of the four data sets examined. Although each engagement starts from an initially neutral position (head on pass, near zero performance index), it can be seen that there is little similarity in outcomes. The initial conditions was taken as the first visual sighting of an opponent (tally). The section performance index was computed as described in Chapter 3 on an engagement by engagement basis and placed in the cumulative tape file.

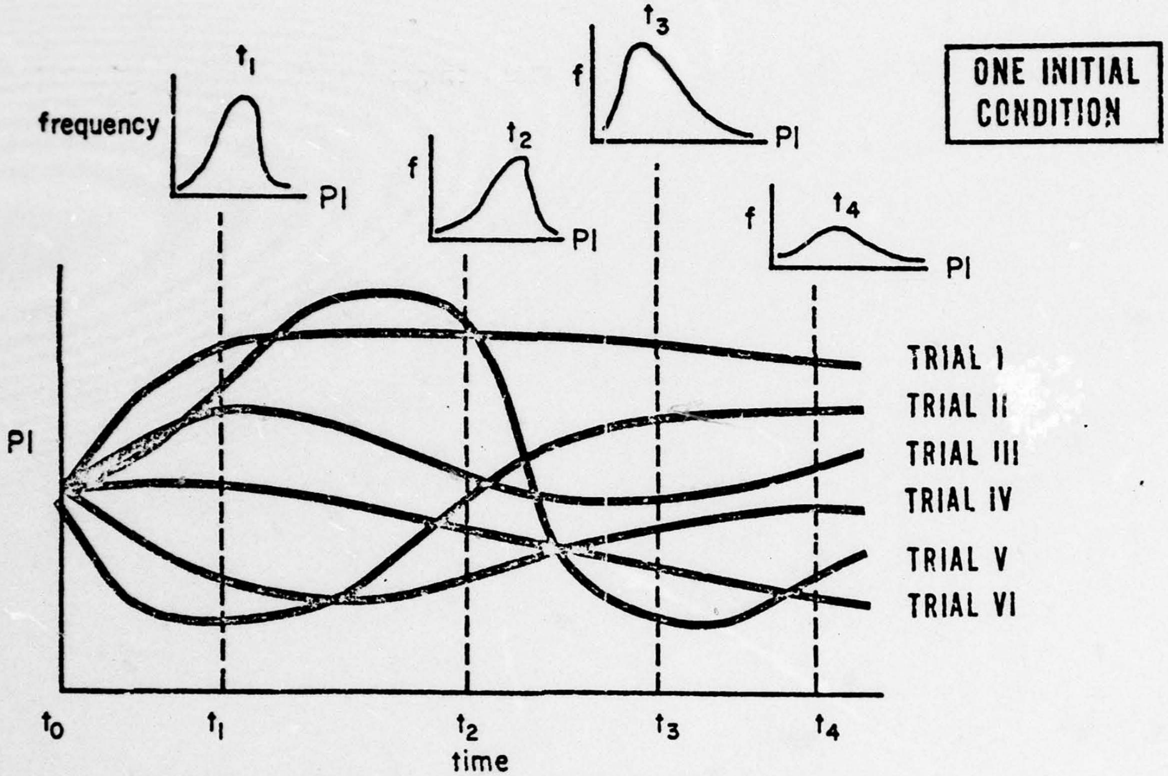
The uniqueness of individual engagements is not unknown, and many analysts feel, not without justification, that air combat data must be handled on an engagement by engagement basis. See, for example, Youngling, et. al. (reference 23). While the analysis of individual engagements is necessary,

techniques for the analysis of the multiple engagement set as an entity are presented.

QUANTIFYING THE RANGE OF EVENTS

Since the uniqueness of each engagement is part of a continuum of events when described by an index of continuous nature, it becomes natural to look for upper and lower bounds, and a characteristic, expected, or most likely value. The upper and lower bounds are computationally set in the structure of the performance index as ±100 since the index was normalized to these values as discussed in Chapter 3. The computation of expected values suggests a statistical descriptor such as a density function, and the physical problem suggests a time sequence of events such that the basic approach to quantifying multiple trials is shown in figure 20. Thus, the family of engagements is taken as a realization of a stochastic process characterizing the relative air combat ability of the opposing forces. The analysis techniques to be described are motivated by this observation. The first application of this observation to the performance index for a family of air combat trials is included in Simpson and Oberle (reference 24).

As shown in figure 20, data is taken at each discrete time slice t_i and a density (frequency of occurrence) function is computed. This set of computed density values



FREQUENCY OF OCCURENCE AS A FUNCTION OF TIME

FIG. 20: STATISTICALLY CHARACTERIZED TEST DATA

will be referred to as the one-dimensional distributions (one-dimensional because there is no cross-correlation in time or value). In general, although not necessary, the experimental trials should begin from a given initial condition, and each engagement should have the same approach to tactics, and set of ground rules for the analysis to have meaning.

THE COMPUTATION OF DISTRIBUTIONS

General

The general problem of estimating an underlying distribution from a given number of random samples has been heavily treated only for the case of known distribution form. These are generally referred to as point estimators and are discussed in detail by Mood, et al, in reference 25. Many attempts have been made to the problem of estimating an unknown underlying distribution, or to verifying the form of an underlying distribution (such as hypothesis testing, see Van Trees, reference 26), but for the most part have required a large sample size for the accurate determination of either distribution forms or values. A computational procedure for evaluating the empirical distribution function has been worked out by Curry and Egbert (reference 27). This work is an application of the work of Parzen (reference 28) and Murthy (reference 29). Figure 21 shows the basic algorithm used in computing frequency distributions as taken from reference 27.

Input:
Random Data = $(x_1, \ldots, x_M)$
Select δ $(0 < \delta < 1)$ $\rightarrow$ $\begin{cases} \delta = 0.010 & \mu(M) > 1 \\ \delta = 0.250 & \mu(M) \leqslant 1 \end{cases}$
Select H $\rightarrow$ $\left\{ H(x) = \exp\left(\frac{-X^2}{2}\right) \right.$

$\downarrow$

Compute:

$$a_{ij} = (x_i - x_j)^2$$

$\downarrow$

Compute:

$$\mu(M) = \frac{1}{M(M-1)} \sum_{i=1}^{M} \sum_{j=1}^{M} a_{ij}$$

$\downarrow$

Compute:

$$\rho = \begin{cases} [\delta \log M + \log \mu(M)] / \mu(M) & \text{if } \mu(M) > 1 \\ \delta \log M / \mu(M) & \text{if } \mu(M) \leqslant 1 \end{cases}$$

$\downarrow$

Compute:

$$\frac{1}{b} = \frac{1}{M(M-1)} \sum_{i=1}^{M} \sum_{j=1}^{M} a_{ij} \exp(-\rho a_{ij})$$

$\downarrow$

Compute:

$$f_M(x) = \frac{b}{M} \sum_{k=1}^{M} H(b(x - x_k))$$

FIG. 21: DISTRIBUTION RECOVERY TECHNIQUE

In figure 21, M random data samples $(X_1 \ldots . X_M)$ are used to compute the estimated frequency of occurrence (f_M) at the point X (which may or may not be a value in the random data set). μ and ρ are computed intermediate steps and δ is an empirically derived constant. H is a weighting function which determines neighborhood of influence of observed samples about the point X. The value of δ was taken as the empirically derived value presented by Curry and Egbert, and the function H was chosen to satisfy the following requirements:

- H(x) is non-negative
- H(x) is even
- $\lim_{||x|| \to +\infty} H(x) = 0$

In the general case H(x) would have to be normalized over the infinite interval, but this requirement was dropped since, in this case at least, it was to be normalized subsequently over a finite interval of ±100 for the performance index. The primary assumption in the development of the distributional estimation is that the unknown underlying distribution have a finite variance. While it is unknown at this point whether the underlying distribution has satisfied this requirement, each of the estimated distributions has thus far met the

requirement (at least to numerical order of magnitude). This requirement is based upon defining the estimated distribution as an unbiased estimator. The fact that the estimated distribution be unbiased is critical to the analysis.

<u>Small Sample Size</u>

One of the most attractive features of the Curry and Egbert methodology is the manner in which it handles small sample sizes. A change in the computation of the exponential multiplier (ρ) occurs when the relative sample size (μ) becomes small. As shown in figure 21 this relative sample size is a measure of the ratio of sample spread to the number of samples:

$$\mu(M) = \frac{1}{M(M-1)} \sum_{i=1}^{M} \sum_{j=1}^{M} (X_i - X_j)^2 \qquad (10)$$

where X_i and X_j are the i^{th} and j^{th} observed samples, respectively. This change in computation allows for a slower fall-off in accuracy when dealing with small sample sizes. Table 5 as taken from reference 27 shows the accuracy of estimation for a normally distributed data set using the mean square error (ε) taken over a 101 point interval from the following definition:

TABLE 5

SENSITIVITY OF FREQUENCY FIT TO SAMPLE SIZE
(reference 29)

Sample Size	Mean Square Error
M	ε
15	3.764×10^{-3}
25	3.903×10^{-3}
50	4.648×10^{-4}
75	1.599×10^{-3}
100	8.843×10^{-4}
250	7.959×10^{-4}
500	3.485×10^{-4}
999	5.340×10^{-5}

$$\varepsilon = \frac{1}{101} \sum_{K=0}^{100} (f_M(y_k) - f(y_k))^2 \tag{11}$$

where

y_k is the computational point

f is the known underlying frequency

f_M is the estimated underlying frequency

and ε is the mean square error.

While table 5 demonstrates an increase in accuracy with sample size, the sample size of 15 yielded a mean square error which is 0.4% of the area under the frequency curve. Curry and Egbert demonstrate frequency fits for several other distributions with the same general trend. The worst fit appears to come from the uniform distribution, which may have an error as high as 15% at 15 samples.

Implementation

The implementation (FREDIS), of the Curry and Egbert methodology was taken as shown in figure 21, with the exceptions noted. Additionally, the following computational algorithms were added to the determination of frequency:

- An integration routine (QSF) for integration of equally spaced functions using a combination of Simpson's rule and Newton's three-eighths rule.

The routine was used for normalization of the frequency function and for computing probabilities of occurrence defined as:

$$P_{<a,b>} = \int_{a}^{b} f(x)dx \tag{12}$$

where $P_{<a,b>}$ is the probability of occurrence of the variable in the interval a,b. If a is the lower bound (L.B.) and b is a variable, then:

$$P(b) = \int_{L.B.}^{b} f(x)dx \tag{13}$$

and P(b) is the cumulative probability of occurrence. This implementation was taken from reference 30 with only slight modifications.

o A moment generating routine (MOMEN) calculating the classical moments of distributed functions with a given frequency distribution. This algorithm is available in most statistics books but was specifically taken from Kendall (reference 31).

This implementation was then tested against three known distributional sets to assure its integrity. Multiple data runs against each of the normal, exponential, and uniform distributions yielded resonable data fits to as low as 10 data samples, which was used as a lower bound for computations. The specific implementation is given in appendix C because of its uniqueness and possible widespread application.

TEST DATA CHARACTERIZATION

The data presented in figure 17 were characterized as shown in figure 20 by the algorithms of figure 21, modified as discussed, and by the control structure shown in figure 22. Figure 22 shows the test data characterization broken down into six computation modules:

- ACE1--control of input, output, vector stripping of matrix data, and some calculational procedures.
- RDEVAL--reads and decodes the cumulative tape file.
- FREDIS--computes the frequency fit as previously discussed.
- QSF--integration routine from reference 30.

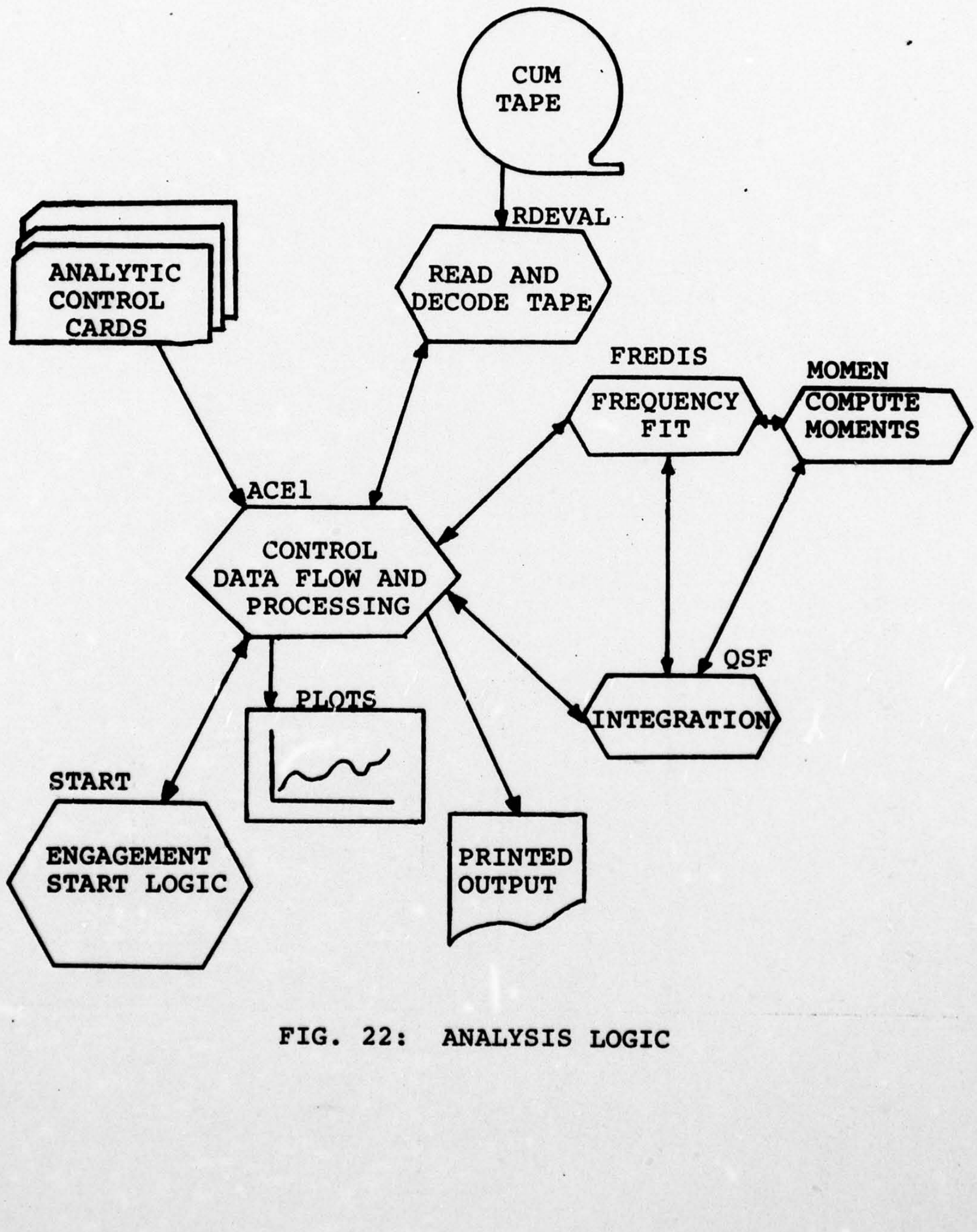


FIG. 22: ANALYSIS LOGIC

- MOMEN--moment generating routine.

- START--a routine which allows the engagement to start at first tally, a specified inter-aircraft range, or a specified time.

Figure 23 shows the frequency data as taken at 10 second intervals over the first 50 seconds of the engagement set. These data are normalized so that the area under each of the curves is identical and equal to 1.0. Figure 24 gives the cumulative probability (integrated frequency data) for the same engagement set. These data, together with their statistical descriptors (mean, variance, higher order moments) will be taken as the characterization of the data set. Use of these data in the detailed analysis of air combat will be given in the next chapter.

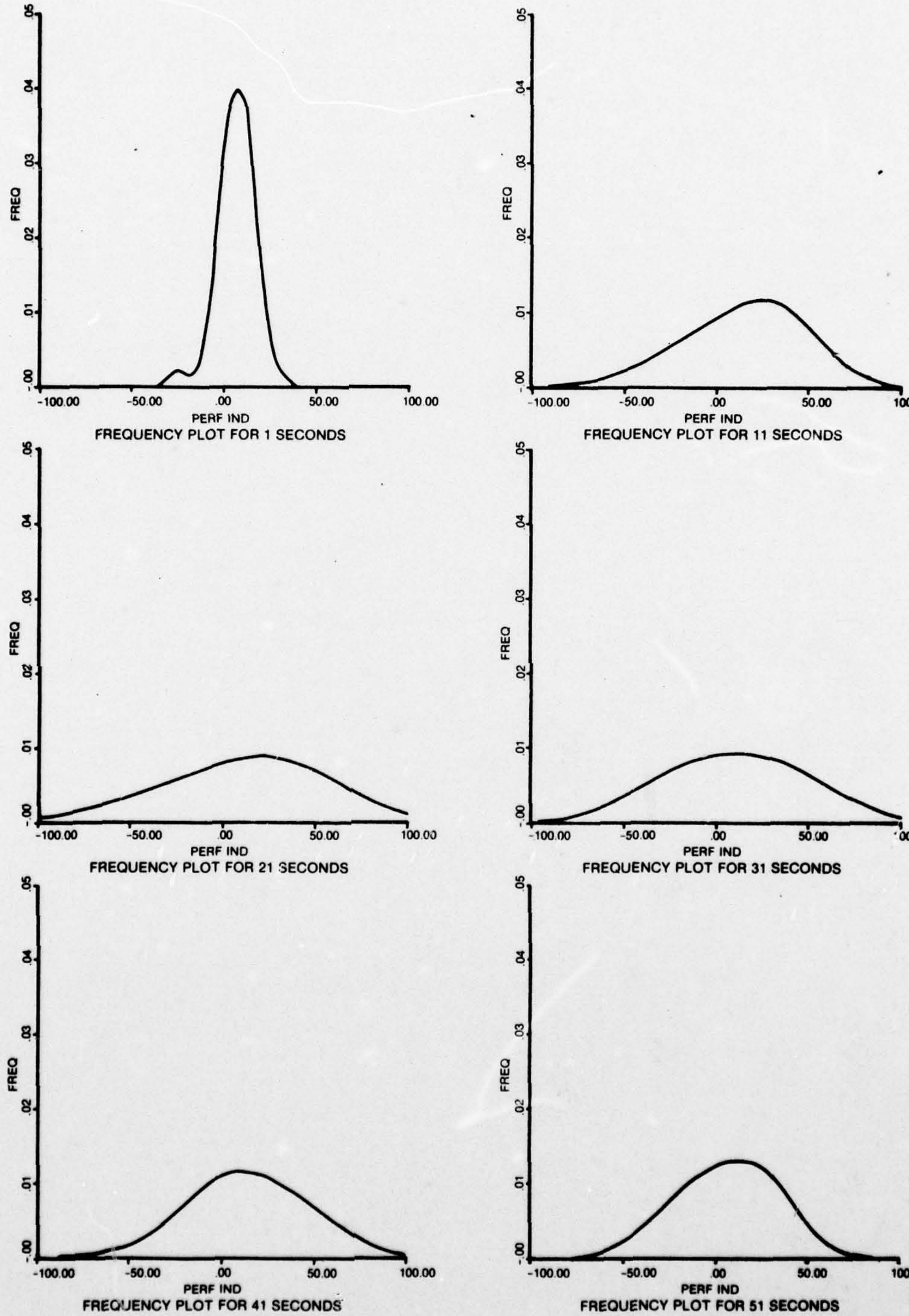


FIG. 23: FREQUENCY CHARACTERIZATION

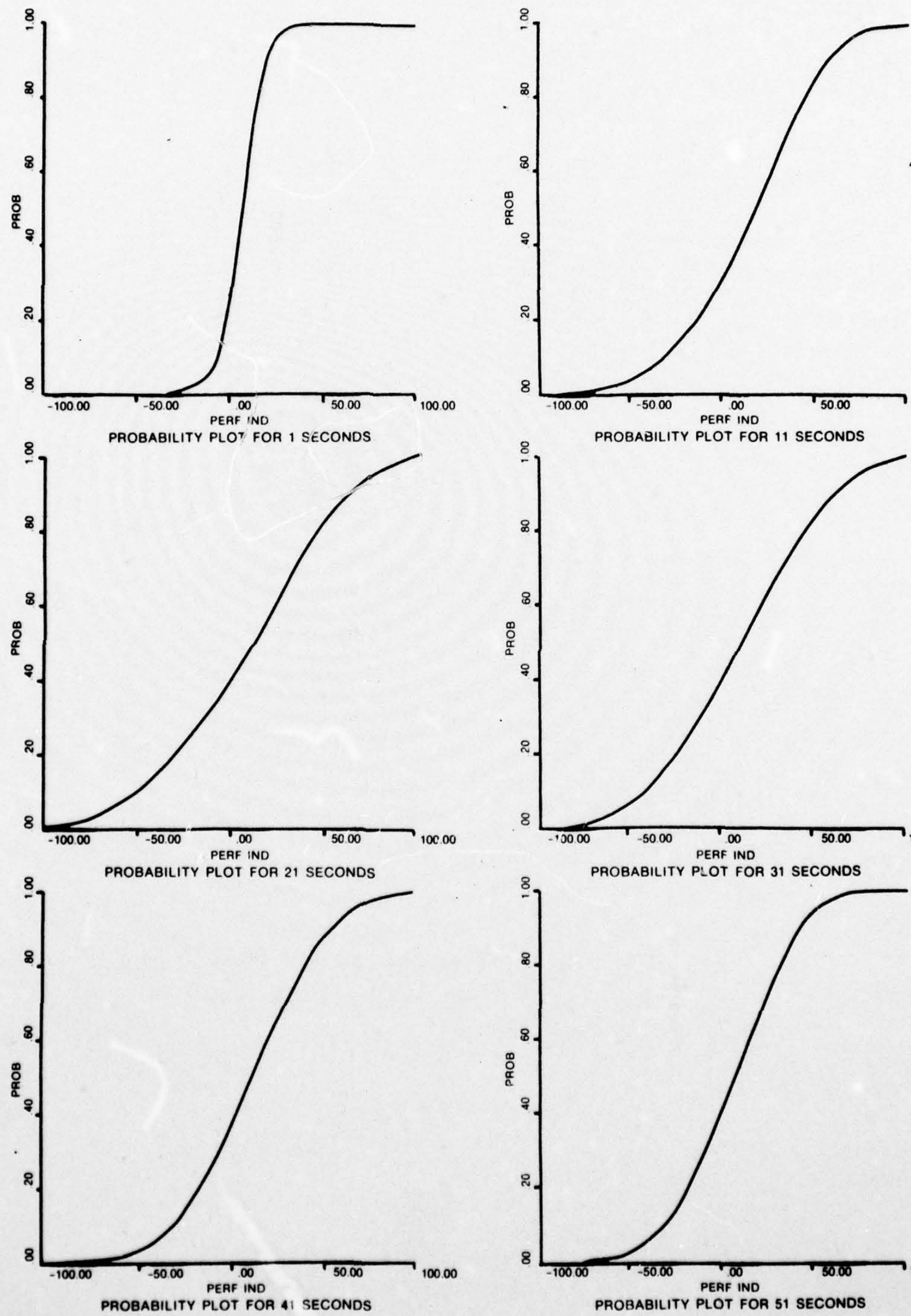


FIG. 24: PROBABILITY CHARACTERIZATION

CHAPTER 5

THE ANALYSIS OF ENGAGEMENT TRIALS

"I believe in perfect laws in a world of existing things,.in so far as they are real, which I try to understand with wild speculation."
-- A. Einstein

INDIVIDUAL ENGAGEMENTS

A large amount of the analysis that can be performed with dynamically interactive systems is in the examination of individual experiments. In the application under consideration, it is especially true in the training and/or testing cycles where the individual engagements may have their largest impact on the system.

In the training cycle, it is important to recognize flaws in the execution of tactics so that they can be corrected or practised until flawless execution can be achieved. Alternatively, effective counter-measures can be recognized or developed, or counters to counter-measures can be developed. Figure 10 of Chapter 3 shows 4 significant break points where detailed analysis should be undertaken. At each of 59, 79, 106 and 131 seconds significant events take place which alter the engagement trend. The first three are directly attributable to actions by either the fighter or the advesary, the fourth break point does not correlate with

AD-A063 760 CENTER FOR NAVAL ANALYSES ARLINGTON VA F/G 15/3
THE ANALYSIS OF DYNAMICALLY INTERACTIVE SYSTEMS (AIR COMBAT BY --ETC(U)
DEC 78 W R SIMPSON

UNCLASSIFIED NL

2 OF 2
AD
A063760

END
DATE
FILMED
3-79
DDC

a specific event but is probably associated with the termination of the engagement.

In the testing cycle, the performance index can be used to evaluate the usefulness of a device or system in a particular situation. Figure 25 shows the computation of the performance index for two similar aircraft, one of which was modified to include a device which had potential application to the air combat environment. The modified aircraft has been placed at a decided disadvantage (in the weapons envelope of the opponent) and the test is to ascertain whether or not the device can assist the pilot in the modified aircraft. As shown in figure 25, a direct correlation between device useage and short term (tactical) gains is evident. In the long term (strategic), gains are not realized. The device, however, may have use in doing such things as breaking a guns tracking solution, or defeating the terminal maneuvering of an air-to-air missile. Additionally, it can be seen that the maximum benefit in this situation is derived some 15-16 seconds after application, leading to the conclusion that another device, or maneuver delivered in conjunction with the first device, but 15-16 seconds later may be even more effective. Care should be taken, however, to not extrapolate to conditions other than those tested as shown in figure 26. In figure 26, the same device is tested in an initially neutral situation with little or no effect on the engagement

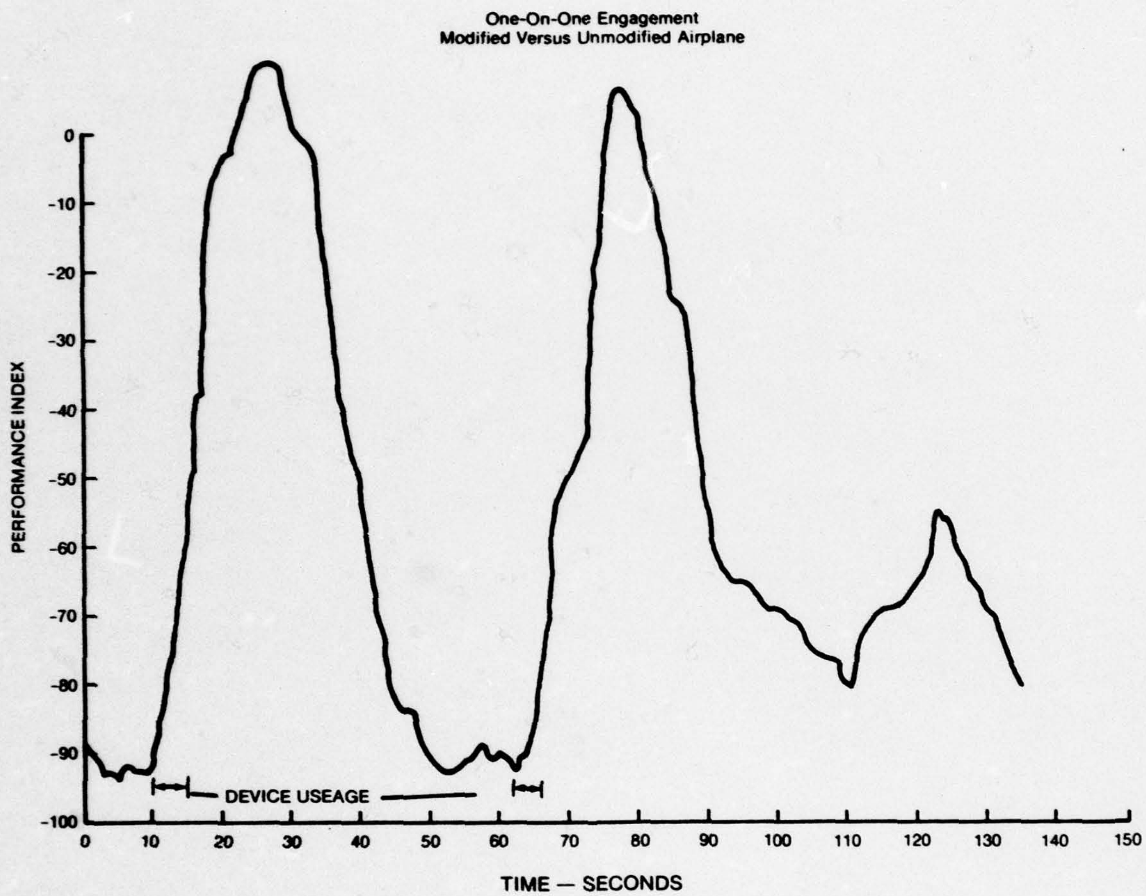


FIG. 25: EXAMPLE 1 FOR DEVICE EVALUATION

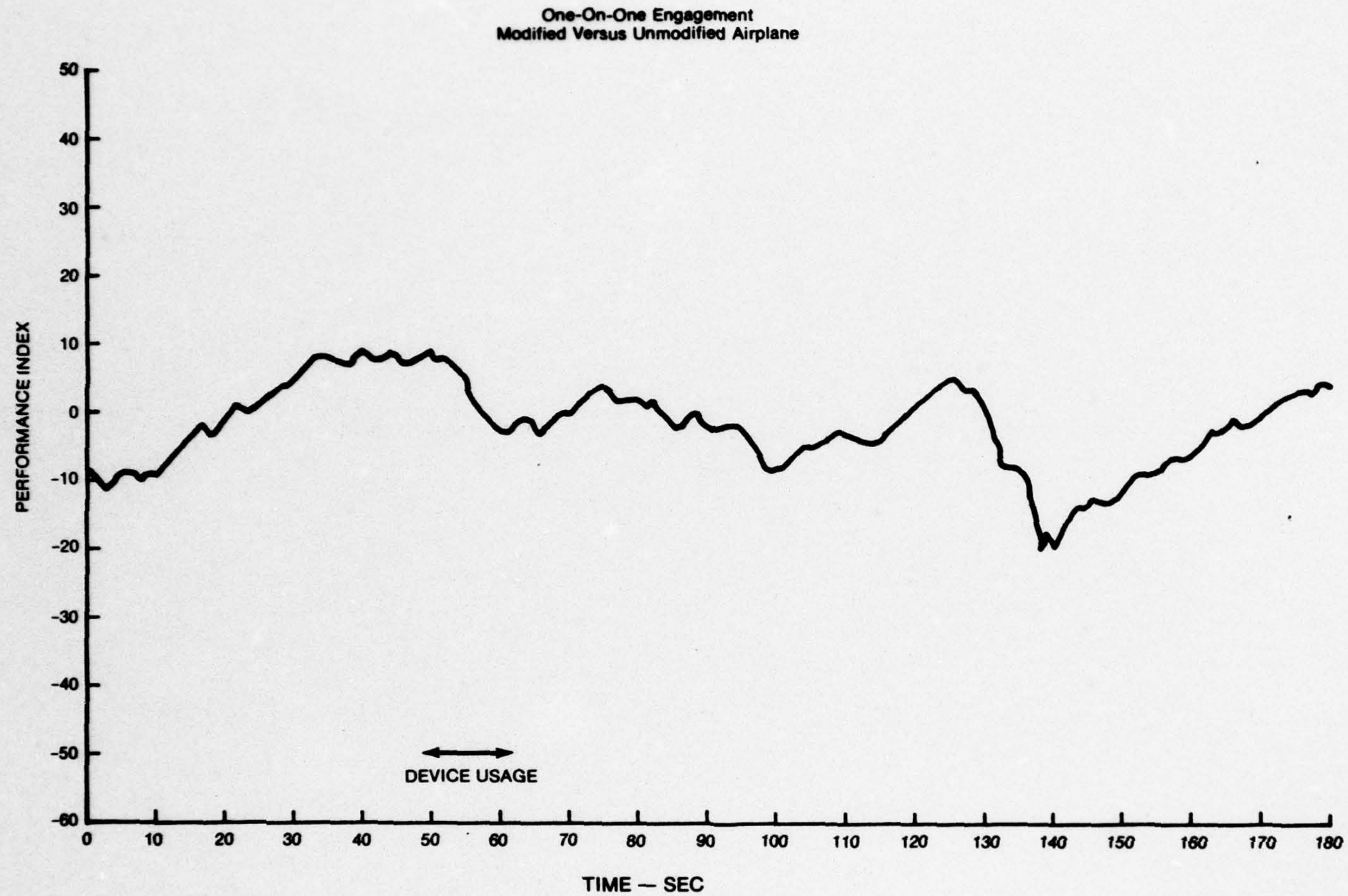


FIG. 26: EXAMPLE 2 FOR DEVICE EVALUATION

dynamics even though, the device was used for some 12 seconds (as opposed to 4-5 seconds in figure 25). The inability to break out of the neutral state (neighborhood of PI = 0) is fairly typical in one on one engagements for well trained pilots in similar aircraft, because there is no performance differential to use to advantage. The device in this case did not evidently supply the needed performance advantage under these circumstances. A large number of the potential changes of the air combat situation which can be brought about by a device or a particular maneuver are shown in figure 27. There are obvious conclusions to be drawn. For example, a device which gives no change in a particular situation must have merit elsewhere or it is not worth its weight or cost. Pure gain and pure loss are obvious, but the intermediate cases of a combination of gains and losses must be examined more closely. A strategic (long term) loss may be acceptable if it provides immediate assistance in a dire situation (tactical gain). For example, the data of figure 25 presents a situation where just such a combination of events would be tolerated. It may also be tolerated if the tactical gain is large enough to provide a weapons opportunity. Conversely, a tactical loss may be tolerated if a strategic gain can be achieved, <u>and</u> the tactical loss is sufficiently small or non-threatening. The individual conditions must determine where and what magnitude losses are acceptable.

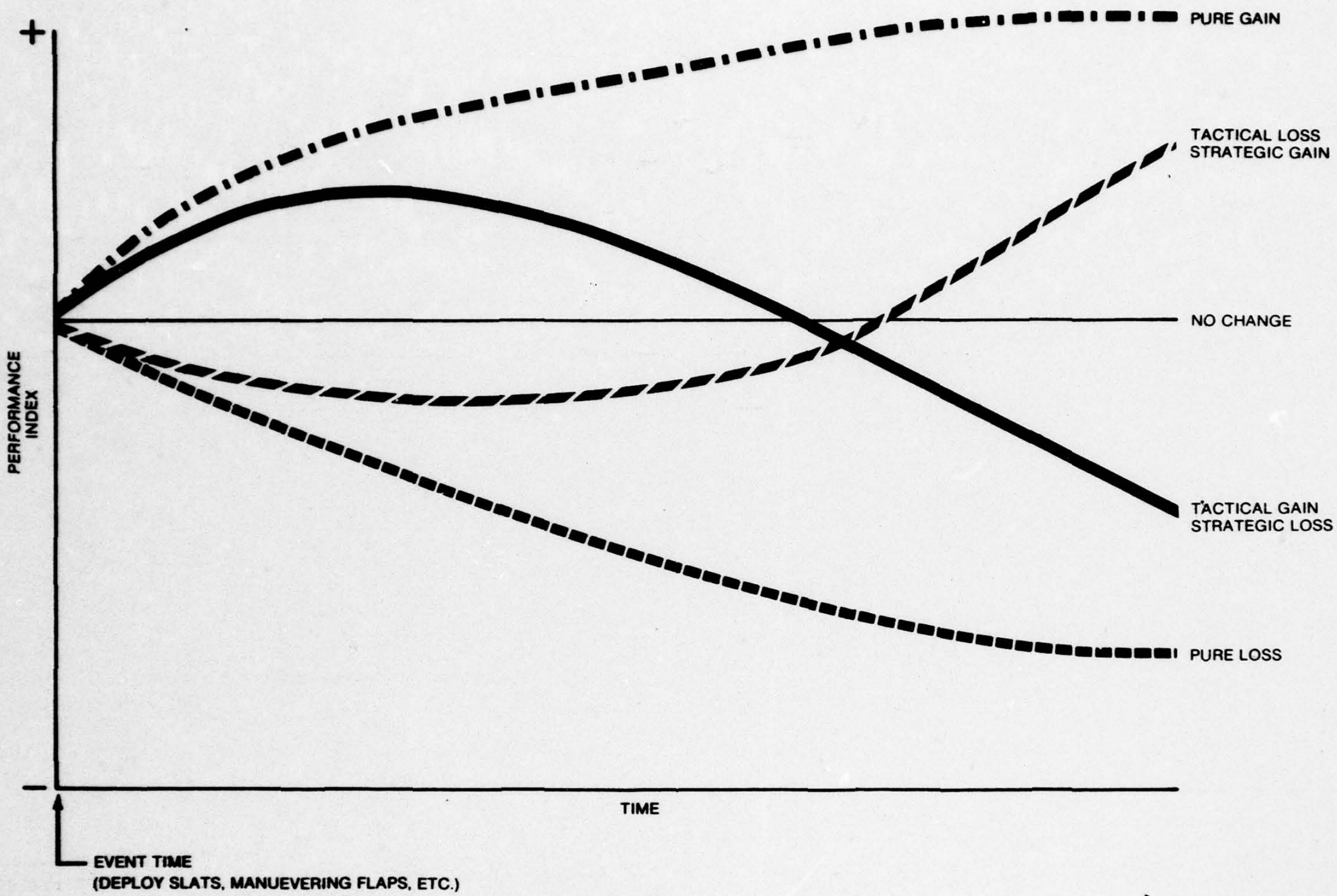


FIG. 27: PERFORMANCE INDEX TIME ANALYSIS

In the test cycle, care should be taken that the evolution of counter-tactics and counter-counter-tactics do not alter the basic conclusions as to the usefulness of the device. For this reason, the analysis of individual engagements should be done in conjunction with an evaluation of multiple engagements.

MULTIPLE ENGAGEMENTS

Data Set Definition

The analysis of multiple engagements is undertaken to ascertain the repeatability of a given set of data and to determine measures of effectiveness which can be used to make decisions about tactics, hardware, or general philosophy of engagement. The first step in the analysis is to define the engagement set as a whole. The following paragraph describes the engagement set shown in figure 17 of Chapter 4.

The data set is defined by a collection of 33 engagements of two fighter aircraft versus one adversary aircraft. The fighter aircraft were of the large (heavy weight), high thrust class with full weapons systems (including on-board radar). The adversary aircraft were of the small highly maneuverable type of a moderate thrust class with a moderate weapons system (no onboard radar). All engagements were started with a neutral (qualitative) initial condition, and duration ranged from just over one minute to just under five

minutes. Tactics employed were predominatly the presentation of one of the fighters as a target to entice the advesary to a vulnerable position with heavy use of free fighter/engaged fighter, but training rules forbade the slashing attack (high energy fight), and all combatants were required to "mix it up and turn with the advesary." Vectoring information (GCI) was supplied to all combatants (fighters or advesaries) for initial run-in. The GCI information was designed to set up a head on intercept at five natucial miles. Inside five nautical miles, or after first visual identification (VID) no further GCI assistance was given. The engagements were allowed to continue without consideration of weapons envelopes until both combatants decided to break-off or a safety violation occurred (such as low altitude or outside of range boundaries).

Analysis of the individual engagements has determined that in 17 of 33 engagements, the fighters had radar contacts prior to termination of GCI, providing an informational advantage. Despite this informational advantage, first tally was achieved about equally by fighters and adversaries. Additionally, the fighters were nearly always at a higher speed on ingress giving them a slight energy advantage. Each engagement was considered to numerically begin with the first tally (either pilot or RIO, fighter or adversary).

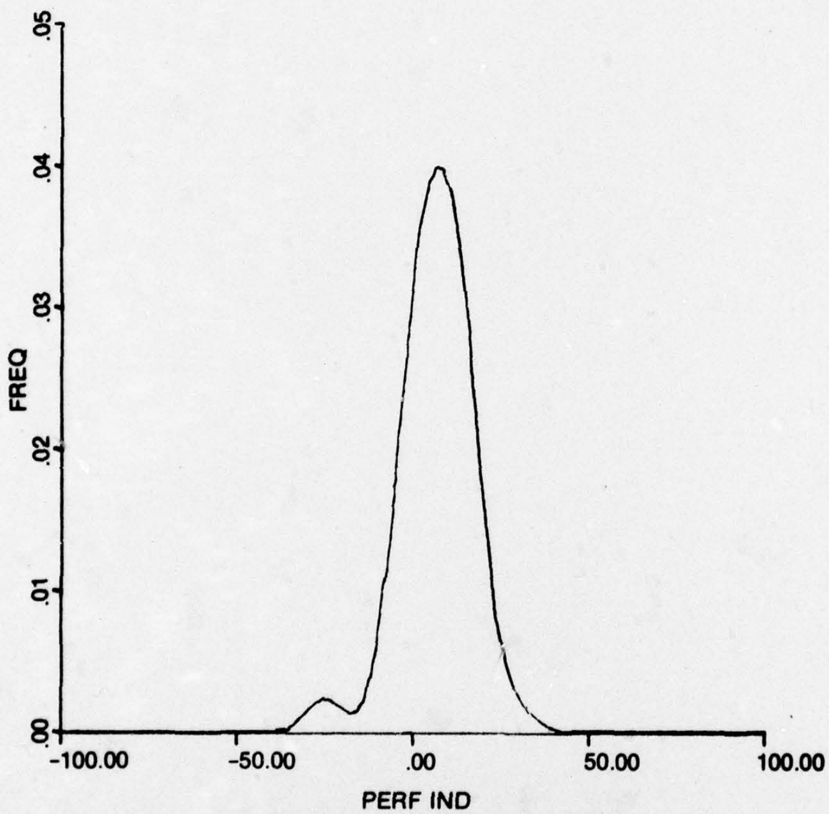
Data Set Characterization

The data set characterization is by the methodology presented in Chapter 4. Figure 28 is the distributional data for the initial starts of the thirty-three engagements and shows that the entire data set comprised a neutral start (±40 in performance index value) with a near normal distribution about the mean (6.34). Table 6 presents the characterization of the full data set in terms of their statistical attributes. The low value of the third moment at the initial condition verifies the near normality of the data set start. Figure 29 shows the frequency fit data at one second intervals for the entire data set.

Analysis of the Data Set Characterization

Figure 29 offers potential for detailed analysis of the engagement set as a whole. The initial drop off in the frequency value indicates a sharp increase in the randomness of events.[4] This would indicate that the choice of time for the initial starts coincided very closely to the initial offensive and defensive maneuvering. The peak of the curve, or the highest naturally occurring frequency stays near the 0 performance index indicating that the two heavier fighters

[4]An increase in the randomness of events is taken to mean a decrease in the difference of probability of occurrence of any range of performance index values -- that is the tendancy toward a uniform distribution which has, at every equal interval, the same probability of occurrence and at every value the same frequency of occurrence.



FREQUENCY PLOT FOR 1 SECONDS

FIG. 28: DISTRIBUTION OF INITIAL STARTS FOR 33 ENGAGEMENTS

TABLE 6

SUMMARY STATISTICS FOR SECTION PERFORMANCE INDEX DISTRIBUTION

Time (sec)	Number of Points	Mean Performance Index	Variance of Performance Index	Third Moment of Performance Index About Mean	Fourth Moment of Performance Index About Mean
1	33	6.34	114.37	-636.67	56159.57
6	33	12.80	462.40	-4176.03	757210.59
11	33	14.57	1151.75	-12832.23	3800686.85
16	33	13.90	1570.17	-19020.63	6318563.84
21	33	10.42	1811.55	-17839.51	8065442.44
26	33	12.29	1782.15	-19620.96	7975665.77
31	33	8.27	1491.78	-1178.67	5558583.45
36	33	8.14	1527.16	-5578.23	6023055.01
41	33	13.02	1120.23	-4447.80	3593418.53
46	33	11.47	1198.91	-5504.27	3974260.30
51	32	5.99	807.96	-4006.78	1734312.40
56	32	3.55	847.04	380.94	1840745.63
61	32	4.27	1000.48	1595.19	2757920.97
66	32	4.97	1249.15	-201.43	4090099.35
71	30	8.30	1702.58	-9678.55	7107357.44
76	30	7.00	1850.01	-7680.61	8024471.00
81	30	4.25	1800.18	-9594.28	7555697.28
86	30	2.31	2300.12	-5018.21	11336444.27
91	29	-0.77	2242.77	1069.86	11057383.18
96	28	-7.02	1766.26	12959.65	7677819.65
101	27	-8.54	1871.21	15561.02	8432997.65
106	27	-8.12	1624.86	14356.95	6497451.97
111	27	-0.96	1797.02	-1663.24	7357124.42
116	27	-4.89	1983.82	8765.20	8980127.66
121	27	-11.43	1540.44	12607.21	5774302.76

TABLE 6 (Continued)

SUMMARY STATISTICS FOR SECTION PERFORMANCE INDEX DISTRIBUTION

Time (sec)	Number of Points	Mean Performance Index	Variance of Performance Index	Third Moment of Performance Index About Mean	Fourth Moment of Performance Index About Mean
126	25	-6.31	1789.32	9495.46	7541745.63
131	24	-2.79	1934.10	5310.66	8574531.13
136	24	-1.51	1957.46	3766.21	8824056.90
141	21	-1.93	1927.95	9173.04	8660194.07
146	19	-3.53	2064.53	9781.27	9692787.76
151	19	-6.13	1768.54	19825.28	7818620.14
156	18	-7.86	1930.58	21118.76	9156490.34
161	18	-9.46	1879.12	19982.05	8677585.29
166	16	-9.20	2209.73	27797.15	11175677.66
171	14	-10.19	2369.51	27960.52	12478168.37
176	14	-13.59	2179.04	31991.96	11230738.18
181	12	-11.99	2422.24	30528.05	12983093.75
186	10	-16.13	1456.25	18236.56	5761935.46

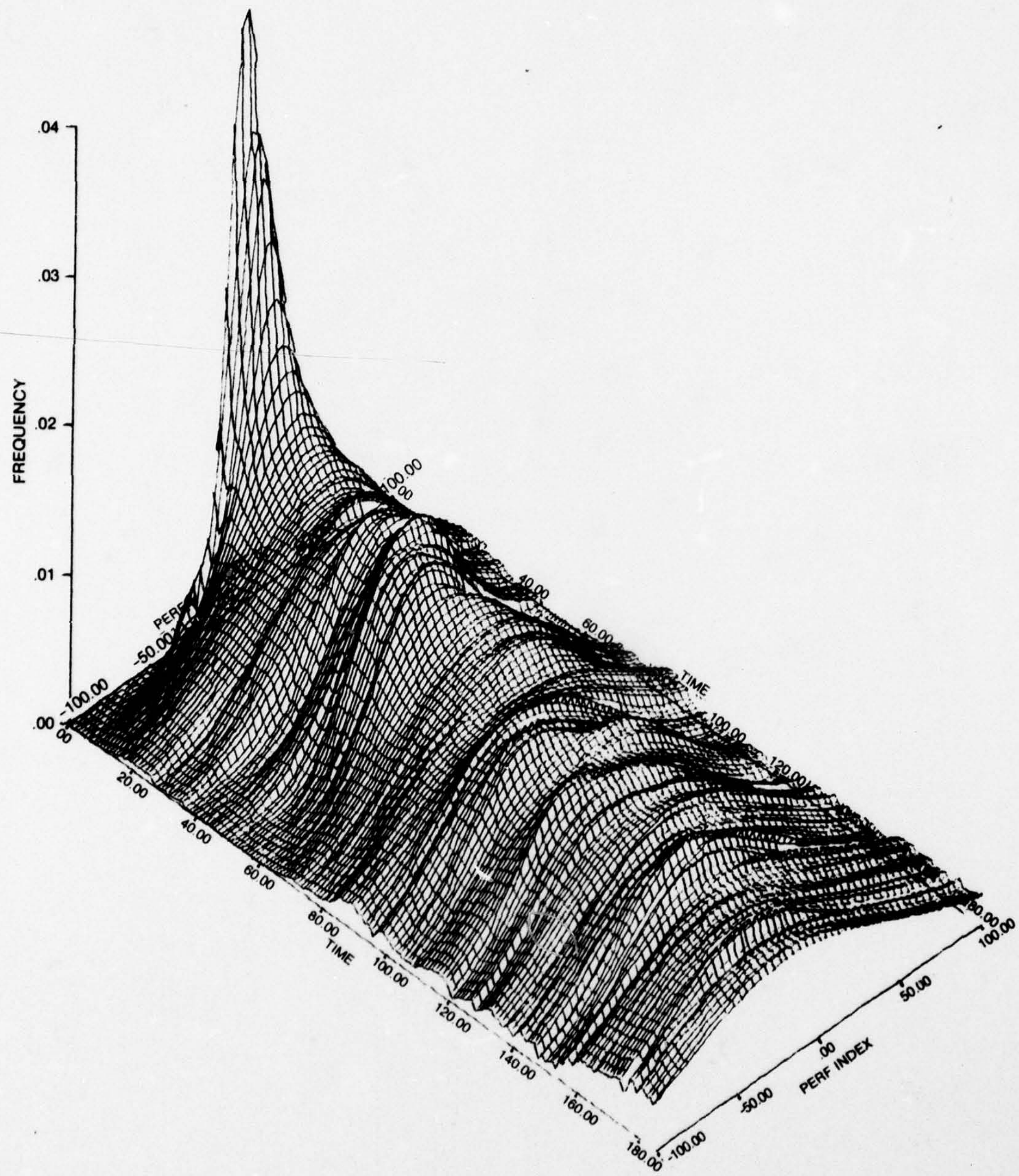


FIG. 29: PERFORMANCE INDEX FREQUENCY CHARACTER BASED ON 33 ENGAGEMENTS

against the one lighter adversary is a fairly equal match-up. This is further demonstrated by the mean value as shown in table 6 which attains an absolute value no greater than 16.13 which is still well within the zone considered neutral. This is also indicated in the rapid drop-off in the peak of the frequency curve with time. If one aircraft section had totally dominated the other, a shift in the value of the performance index at the peak with little drop-off in the frequency would be expected.

The decrease in peak frequency (increase in random events) continues throughout time as demonstrated by figure 30 which shows the degradation to a near uniform distribution, but with what non-uniformity there is skewed in favor of the adversary aircraft. This advantage to the adversary is demonstrated in figure 29 by viewing the frequency value in time along the -100 axis of performance index (optimal weapons delivery for the adversary). These values are cross-plotted in Figure 33. The frequency of occurrence of the optimal position for the underlying distribution is immeasureably small until some 70 seconds into the engagement. At this point, the frequency of occurrence increases in magnitude with time (with some oscillations). Examination of the +100 axis of performance index (optimal weapons delivery for the fighter section) can be analyzed by figure 31. These values are cross-plotted in Figure 34. In figure 31 it is shown

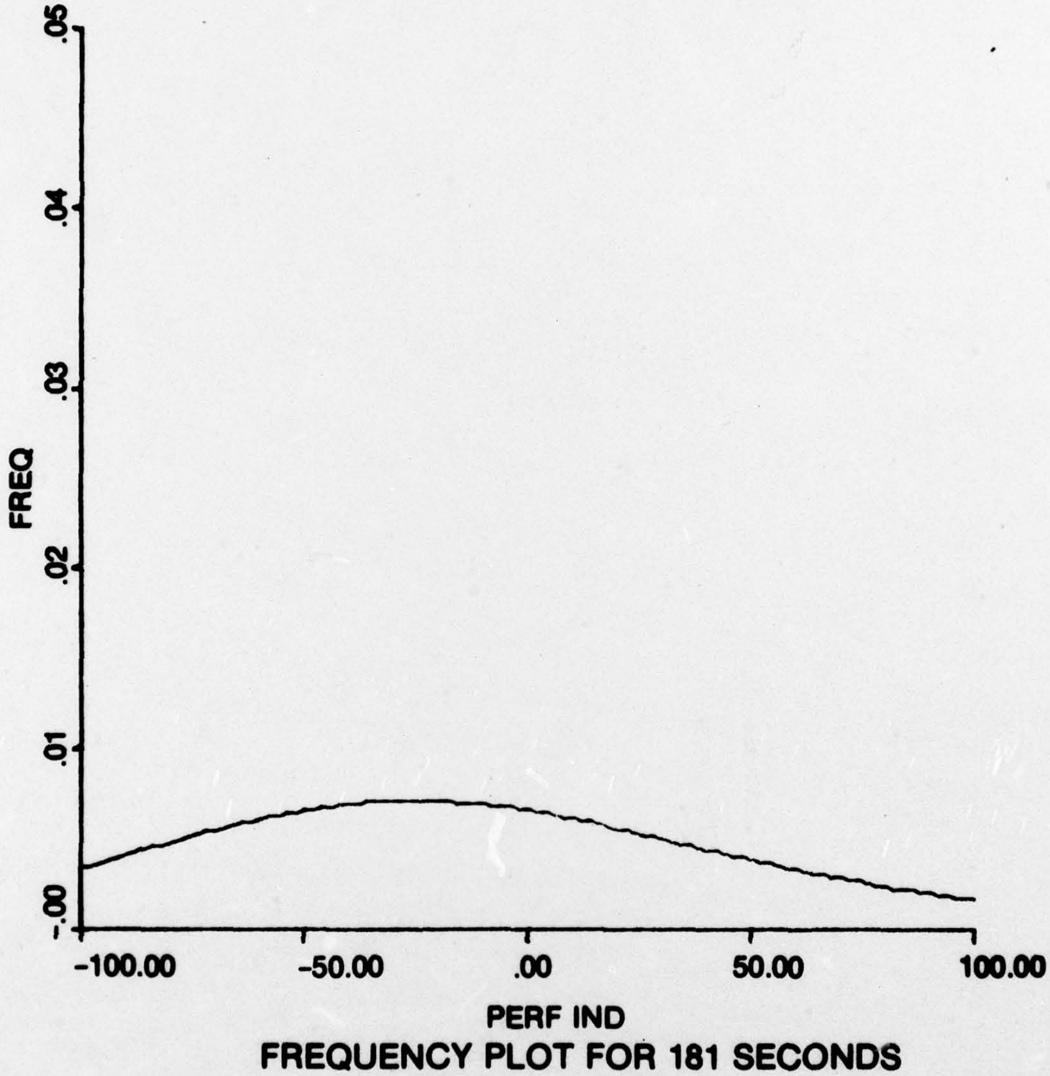


FIG. 30: DISTRIBUTION OF EVENTS AT 181 SECONDS

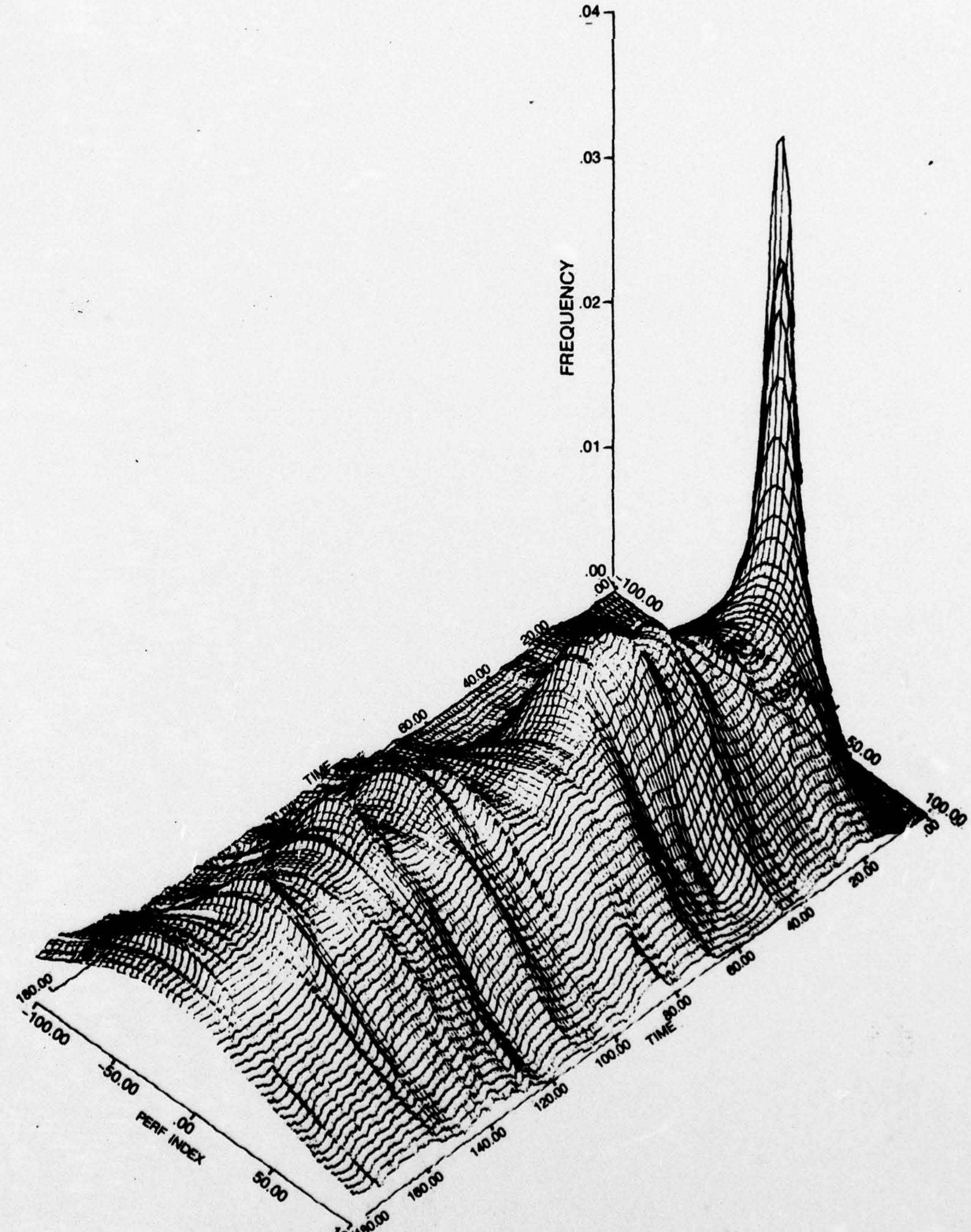
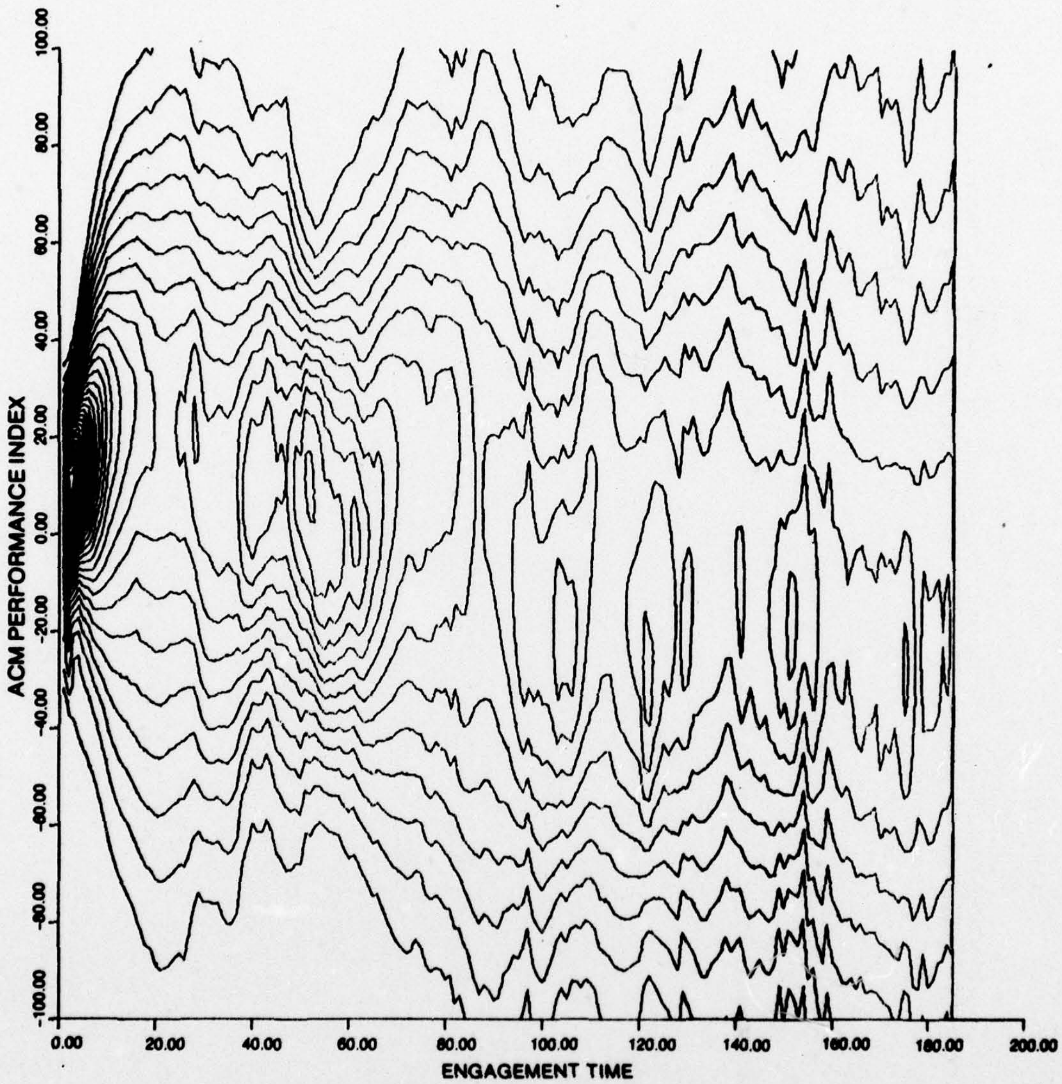


FIG. 31: ACM FREQUENCY CHARACTER

that measurable frequencies occur much earlier in the engagement (10-30 seconds) indicating an early advantage for the fighter section. Although measureable frequencies are present after 70 seconds, they do not rise dramatically as in the case of the adversaries.

These items can be better illustrated by the use of figure 32. Figure 32 is a contour plotting of the three dimensional figure shown in figures 29 and 31. The contours are taken at 0.001 intervals in the frequency values. The figure is generated by cutting figure 29 with planes parallel to the base plane. The lines then, represent constant frequency at 0.001 intervals. In moving from either the upper left or lower left corner, to the center right of the figure the first curve encountered is a frequency of 0.001 the second curve is 0.002, etc. A dramatic demonstration of the difference between the early time of the engagement set and later time is shown by noting that the frequency value of 0.001 first occurs at about 20 seconds for the optimal weapons delivery for the fighter section (where the 0.001 curve intersects +100 in performance index), but is delayed to around 84 seconds for the optimal weapons delivery for the adversary (where the 0.001 curve intercepts -100 in performance index). In constrast, the frequency value of 0.002 first occurs at about 157 seconds for the adversary (where 0.002 intersects -100 in performance index), but does not occur in the data



FREQUENCY PI CONTOURS
EVERY 0.001 IN FREQUENCY

FIG. 32: FREQUENCY CONTOURS

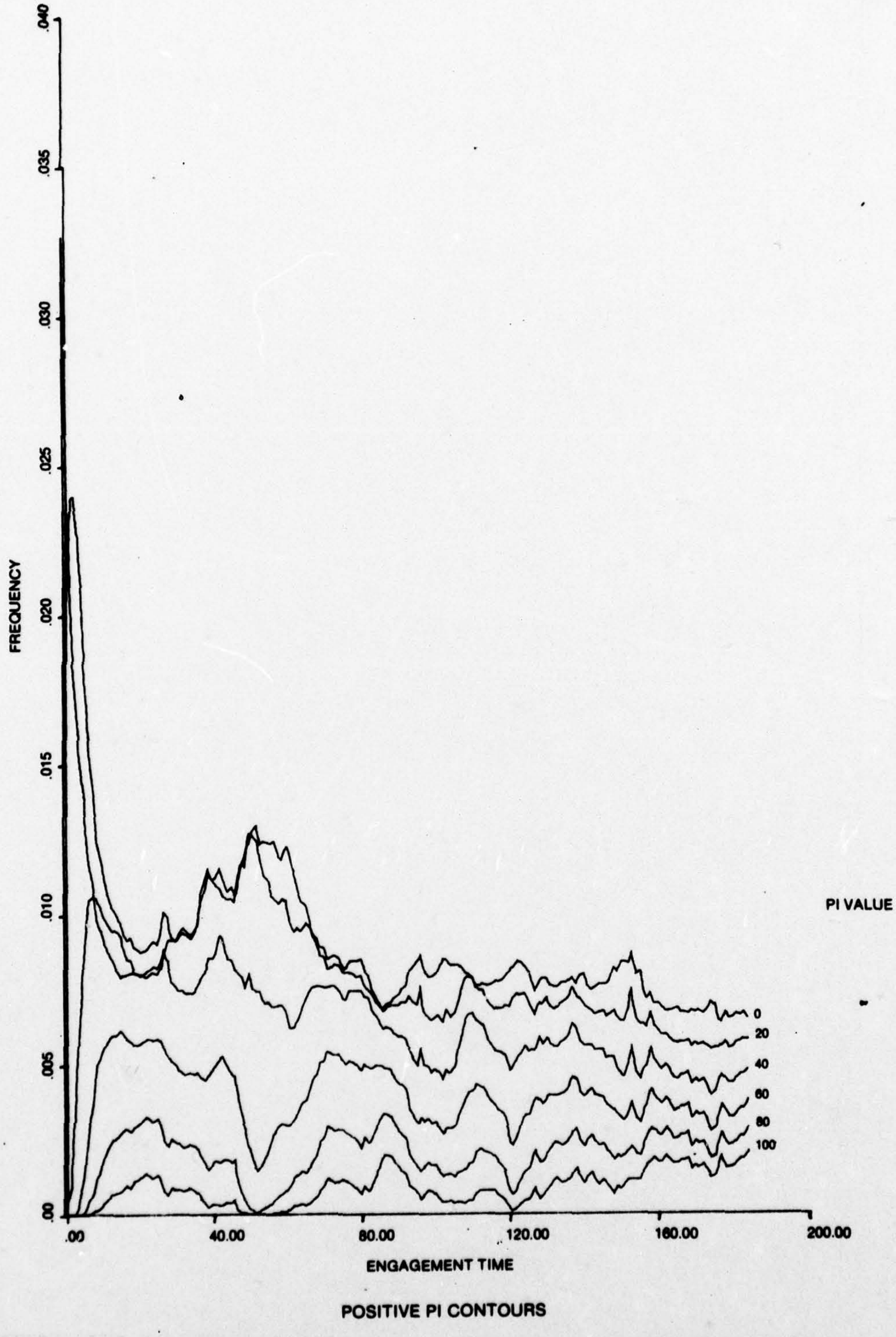
set for the fighter section (0.002 does not intersect +100 in performance index). The figure can also be used to show the variation in the peak frequency location with time by tracing the highest naturally occurring frequency (the smallest closed figures) in time. It can be seen that this moves first in favor of the fighter section (more positive PI) and later moving in favor of the adversary, finally crossing PI = 0 at about 90 seconds. Table 7 gives the numeric values of performance index at peak frequency at 5 seconds intervals. The radical dropoff in peak frequency is also apparent. For comparison, a uniform distribution over the interval <+100, -100> would have a constant value of 0.005.

Figures 33 and 34 show contours taken at positive and negative performance index values respectively. These figures tend to reinforce the points previously stated. For example, the radical drop-off in frequency in time of the initial portion of the engagement. The long term drop-off in the frequency of the neutral values (±20 in performance index), and the long term rise in weapon opportunities (-80 to -100 for the adversary, and +80 to +100 for the fighter section) is a natural, expected, and believable outcome. The figures also point out the major trend reversal which occurs at about 50 seconds, where the incidence of neutral increases and the

TABLE 7

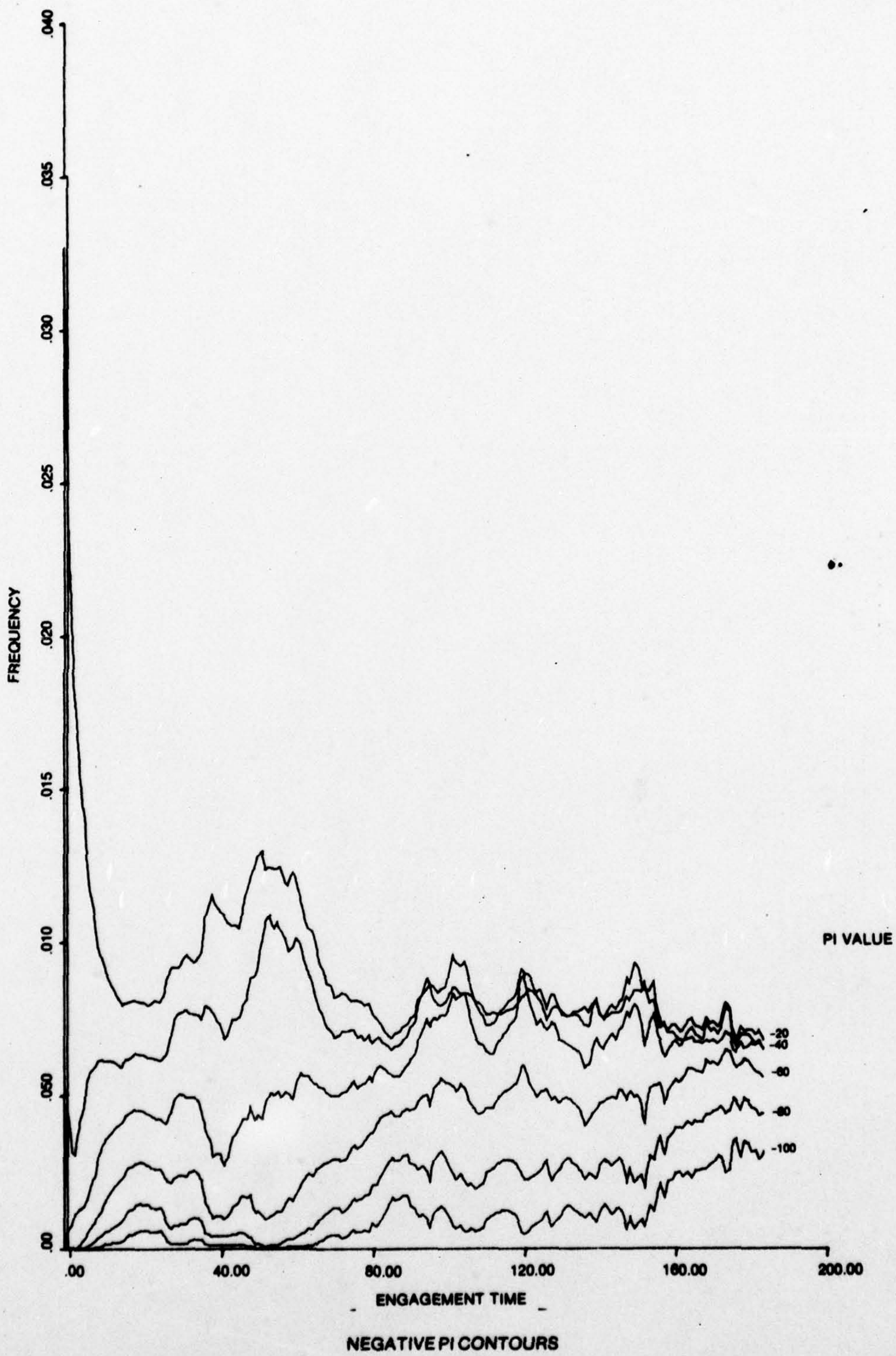
PERFORMANCE INDEX VALUES AT PEAK FREQUENCY (Mode)

Time (sec)	Performance Index (rounded)	Peak Frequency
1	6	.03991
6	16	.02034
11	24	.01190
16	28	.00997
21	22	.00887
26	22	.00907
31	9	.00936
36	9	.00976
41	10	.01162
46	12	.01115
51	12	.01309
56	0	.01238
61	0	.01229
66	5	.01043
71	14	.00895
76	12	.00827
81	18	.00854
86	10	.00701
91	2	.00734
96	-15	.00888
101	-17	.00857
106	-22	.00941
111	13	.00800
116	-14	.00790
121	-28	.00931
126	-16	.00846
131	-10	.00796
136	-5	.00780
141	-12	.00830
146	-12	.00786
151	-18	.00937
156	-18	.00875
161	-16	.00731
166	-23	.00757
171	-25	.00743
176	-28	.00796
181	-26	.00717



POSITIVE PI CONTOURS

FIG. 33: PERFORMANCE INDEX CONTOURS FOR POSITIVE VALUES



NEGATIVE PI CONTOURS

FIG. 34: PERFORMANCE INDEX CONTOURS FOR NEGATIVE VALUES

weapon opportunities fall to near-zero. These would tend to suggest a lull in the activity of the pursuit game, but is as of yet unexplained.

SUMMARY ANALYSIS OF MULTIPLE ENGAGEMENTS

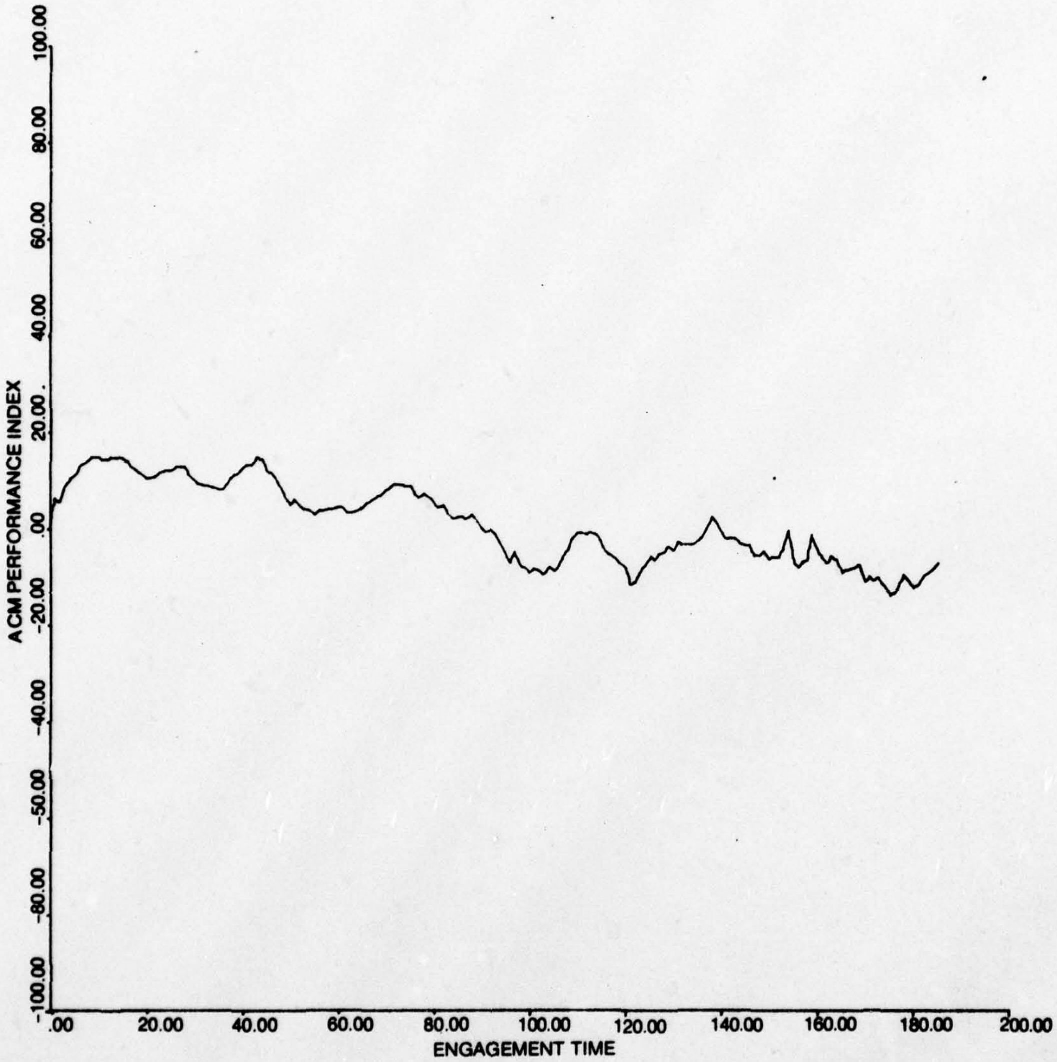
The data set, thus characterized, can be summarized as initially neutral with an immediate swing in favor of the fighter section. This is lost at some time later in the engagement (around 90 seconds) to the more maneuverable adversary. The initial swing in favor of the fighter is the outcome of the informational (onboard radar) and energy advantages. The informational advantage disappears very quickly with the aircraft moving into the visual arena, and the energy advantage disappears more slowly, but just as surely in the turning with the more maneuverable bogey. As these two advantages disappear, and section coordination breaks down, the bogey is selectively engaging the most vulnerable fighter. The early advantage is extremely important to the fighter section in that it will get the largest number of early shot opportunities and thus terminate those engagements quickly. This suggests an early aggressive approach by the fighter section. The long term loss is significant in that the point at which a decision must be made to continue or break off the engagement can be identified (around 90 seconds). For the adversary, of course, the analysis is

different. It must be his job to minimize the effect of the early informational advantage, and negate the energy advantage as quickly as possible. He must additionally survive the early portion of the engagement to take advantage of later opportunities. This suggests an early conservative approach for the adversary.

If these suggestions cause changes in the tactical approach to the engagement, then, of course, the engagement set should be retested to make certain that outcomes are not altered.

THE DEVELOPMENT OF MEASURES OF EFFECTIVENESS

Because of the large number of considerations, and the extreme number of computations and available numbers, it is important to develop reasonable measures of effectiveness (MOE) which reflect the above analyses. The tendancy in most analyses involving statistical data is to examine the central measures (expected values) such as mean, mode, median, etc. Several of these have been previously examined, and as shown in tables 6 and 7, these do offer some of the analysis information available. For example, in table 6 and figure 35 the mean value changes sign at around 90 seconds this also represents a transition in the value of the mode (table 7). This time was previously identified as the point at which the fighter section loses its initial benefit.



MEAN VALUE

FIG. 35: MEAN VALUE

These central measures, however, are of lesser benefit because they are well within the neutral values of performance index values, and hence it is hard to assign engagement outcome significance to them. The really significant measures occur in the outer or limiting values of the performance index which represent the weapons envelope data. These values can be related directly to the terminal phases of the engagement. These measures can also be used for direct comparison in data sets.

The first step in the computation of these measures is to define an area of interest. This region should correspond roughly to the weapons carried by the aircraft and may in general be indicative of but not directly related to weapons envelope. The values chosen for this analysis were taken arbitrarily as 80 to 100 for both the fighter (positive PI) and adversary (negative PI). The value chosen may in general be different for the given combatants. For example, if an F-4 with AIM-9H weapons were engaging a MIG-15 with only short range guns, the values of interest might be 70 to 100 for the fighters and 90 to 100 for the adversary. Chosing the same region for fighters and adversary is equivalent to stating that the weapons have similar capability.

These value ranges represent the tails of the distributional data (see figure 30). By integrating the density

value over this range, a probability that the value lies in the region of interest is obtained as detailed in Chapter 4. This probability taken over a unit time value represents the expected time of occurrence of this range of values of the distribution in that time increment. This is then integrated over time to give a cumulative time in the region of interest as shown in figure 36. If the region of interest is the weapons envelope, then the integrated value represents a cumulative weapons envelope time as a function of time. The difference between the fighter sections expected time accumulation and the adversary time accumulation will then yield the measures of effectiveness as shown in figure 37.

As shown in figure 37, the fighter section accumulates more weapons envelope time than the adversary over the first 89 seconds of the engagement. This region has been labeled as fighter section dominance and can be quantified by either the integral of the function (area under the curve-in this case approx 44.5 sec^2) or by noting in figure 36, that there is some time lag between the fighters accumulation of envelope time and the bogeys. This would be represented by the area between the two curves over the first 89 seconds (same value). The peak of the curve is labeled the tactical decision point and represents the point at which a decision to pursue the engagement or BUGOUT should be made. The

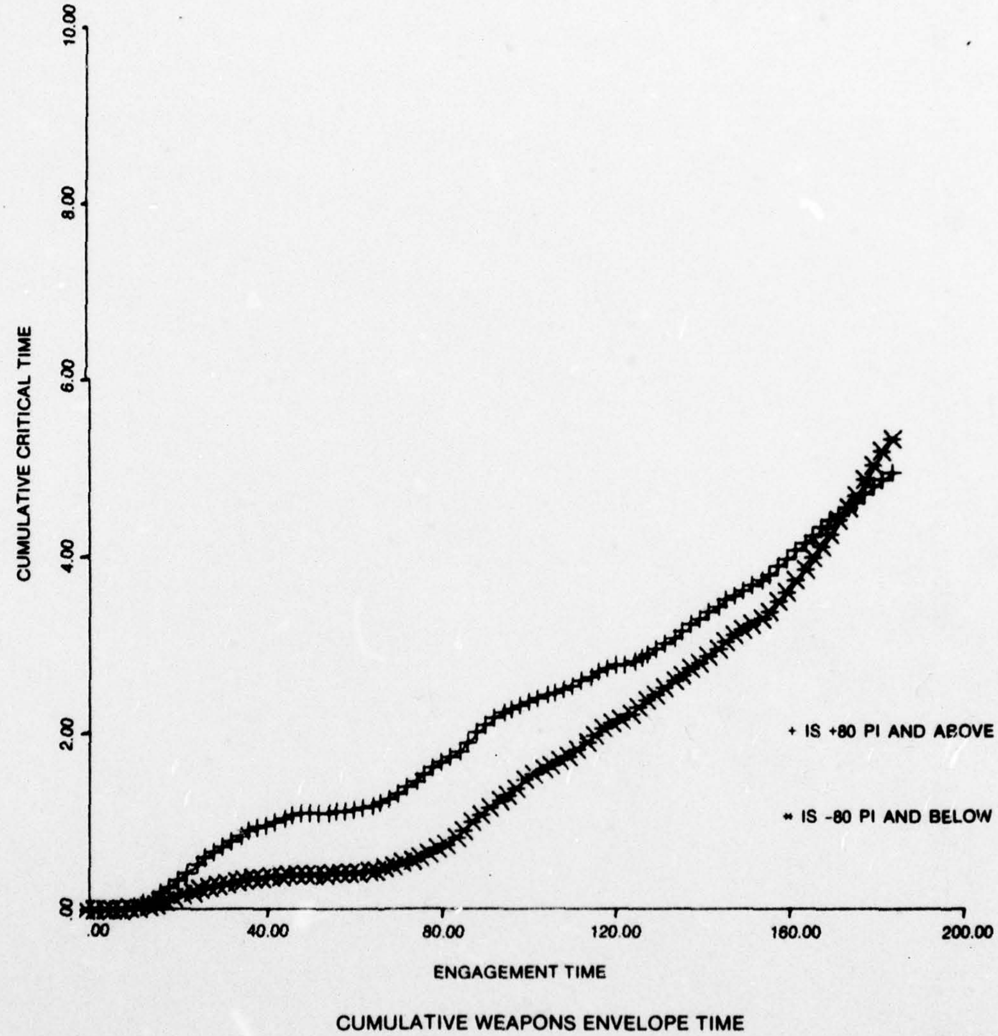


FIG. 36: CUMULATIVE WEAPONS ENVELOPE TIME

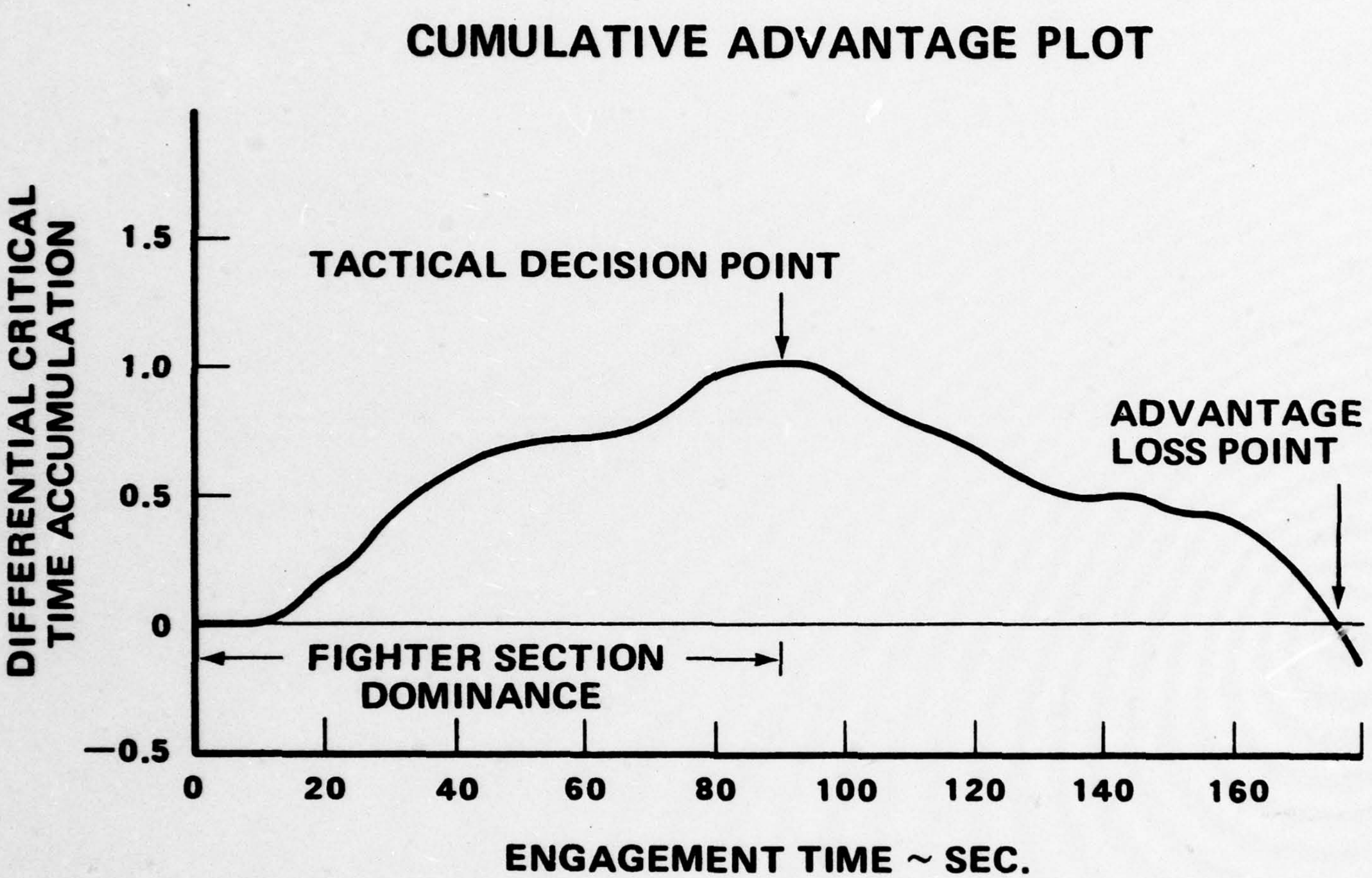


FIG. 37: CUMULATIVE ADVANTAGE PLOT

The final measure given by figure 37 is the point at which the early advantage is completely dissipated by the adversary. This is labeled the advantage loss point and occurs some 174 seconds into the engagement for the sample data set. These measures can then be used to describe the engagement set using fewer numbers for analyses. Chapter 6 will present a specific example of their use.

CHAPTER 6

THE SENSITIVITY OF ENGAGEMENTS TO SMALL CHANGES IN INITIAL CONDITIONS

"The purpose of computing is insight, not numbers."
-- R.W. Hamming

METHOD OF ANALYSIS

The data set presented in Chapter 4, figure 17 was analyzed to ascertain the sensitivity of air combat engagements to small changes in initial conditions. With the performance index, neutral is given by a value of 0.0 giving easy numerical definition to small changes in the initial start. The analysis method employed was to fractionate the data set and then proceed with the computation of the measures of effectiveness described in Chapter 5. The boundaries of fractionation were chosen to yield a workable data set, and complementary sets were not available in each instance, so that complementary sets were not examined. Figure 28 of Chapter 5 gives the distribution of initial starts. It can be seen that the entire data set comprises a neutral start with mean value PI equal to 6.34 (table 6) and standard deviation of 10.69 (square root of variance table 6). Figures 29 and 31 show the stochastic representation of the full data set taken as the representative density function

in time. The data set was then fractionated according to two criteria:

- Initial engagement start as shown in table 8.
- Initial engagement trend as shown in table 9.

In the former, two measures of effectiveness were used in the analysis:

- The tactical decision point (the point at which fighter dominance was lost).
- The advantage loss point.

These measures were computed by accumulating expected values of weapons envelope time. The weapons envelope time was computed via a double integration of the density function in the applicable region of weapons delivery as described in Chapter 5. Figure 37 shows the cumulative advantage plot for the baseline case indicating both measures of effectiveness.

In the latter case, these measures of effectiveness were used, and one additional measure was added to account for the magnitude of the fighter dominance in the opening period of the engagement. The measure was taken as the time it takes the adversary to reach 10% of the value of expected envelope time that the fighter section has accumulated. This number

TABLE 8

DATA SET FRACTIONATION
INITIAL START

Criteria	Number of Engagements	Description
+5 to -100	15	disadvantaged
+100 to 0	27	advantaged
+100 to -100	33	base line
+10 to -10	20	neutral
+5 to -5	13	tight neutral

TABLE 9

DATA SET FRACTIONATION
ENGAGEMENT TREND

Criteria	Number of Engagements	Description
Positive PI slope 1st 6 points	15	Heavily favored trend
Positive PI slope 1st 4 points	20	Moderately favored trend
Positive PI slope 1st 2 points	22	Lightly favored trend
All engagements	33	Baseline

represents the time it takes the adversary to recover from the initial engagement trend.

INITIAL START ANALYSIS

The measures of effectiveness for the fractionated data sets listed in table 8 are presented in table 10. Small changes in the initial conditions were obtained, by deleting engagements from the data set. As shown in table 10 to get from the base case to the advantaged case, four engagements were dropped because their initial performance index value was negative. The effect on the distributional data was to increase the mean value of the initial performance index to 9.33 from 6.34 giving a quantitative shift of 2.99. Each of the data sets presented, approximated a near normal initial condition with mean value as presented in table 10. In the instances of the disadvantaged and tight neutral starts, not enough data remained to carry the computation all the way to the advantage loss point. Table 10 indicates a very low sensitivity to small changes in initial starts in both the tactical decision point and the advantage loss point. Specifically, the flat slope in the neighborhood of the tactical decision point (see figure 37) negates the differences shown in table 10, while the advantage loss point is within 4 percent where there is sufficient data to compute the figure. This lack of sensitivity to small changes in the

TABLE 10

INITIAL START MOEs RESULTING FROM FRACTIONALIZATION

Description	PI Range	Mean Initial PI Value	Number of Engagements	Tactical Decision Point	Advantage Loss Point
Base	+100 to -100	6.34	33	89 seconds	174 seconds
Advantaged	+100 to 0	9.33	27	79 seconds	175 seconds
Disadvantaged	+5 to -100	-1.48	15	93 seconds	not enough data
Neutral	+10 to -10	2.57	20	93 seconds	181 seconds
Tight neutral	+5 to -5	0.72	13	87 seconds	not enough data

neighborhood of neutral is surprising in that it indicates that a large change is needed to affect engagement outcomes when they are started from an initially neutral condition. This conclusion, of course, must be limited to the aircraft, tactics, and pilot proficiency levels embodied in the test sample, and to the near neighborhood of a neutral start. It was shown in Chapter 5 that in radically different initial conditions conclusions may be altered.

ENGAGEMENT TREND ANALYSIS

The measures of effectiveness for the fractionated data sets listed in table 9 are presented in table 11. Changes in engagement trend at the initial onset of the analysis (first tally) were obtained by deleting those engagements which did not meet the criteria for a positive upward slope on the performance index measure over a given number of data points. Only one of the data sets fractionated this way showed a cross-correlation with initial start as indicated by the mean initial PI value in column four of table 11. It can be noted that the positive engagement trend radically shifts forward the tactical decision point. This would correlate well with the pilot recognition of a developing trend, and the subsequent much harder engagement being pursued by the fighter aircraft to convert to a quick victory. This level of effort seems almost binary in that

TABLE 11

ENGAGEMENT TREND MOEs RESULTING FROM FRACTIONALIZATION

Description	PI Criteria	Number of Engagements	Mean Initial PI Value	Time before adversary reaches 10% of fighter value	Tactical decision point	Advantage loss point
Base	All slopes	33	6.34	0 seconds	89 seconds	174 seconds
Lightly favored trend	Positive slope: 1st 2 points	22	6.97	18 seconds	56 seconds	not enough data > 170 seconds
Moderately favored trend	Positive slope: 1st 4 points	20	1.52	12 seconds	56 seconds	not enough data > 162 seconds
Heavily favored trend	Positive slope: 1st 6 points	15	6.59	71 seconds	56 seconds	not enough data

the tactical decision point is invariant when any favorable trend is isolated. The harder fight, of course, causes a much quicker depletion in fighter energy which gives a quicker emphasis to the adversary maneuverability and hence the earlier tactical decision point. The forward shift in the tactical decision point is a direct consequence of the earlier engagement dominance as seen by the time before the adversary begins to accumulate significant weapons envelope time. The counter-trend to this noted in the moderately favored trend is discounted for two reasons. The first is that the slope of the cumulative advantage plot is very shallow over the first few seconds of the engagement set as shown by figure 37. Secondly, the large shift in the mean initial performance index value may make this data subject to cross-correlation effects which have not been explored because of the limits of sample size.

SENSITIVITY SUMMARY

The engagements analyzed would indicate that in the two-versus-one air combat arena, the initial engagement position is much less significant than the engagement trend. The latter, while decreasing the time before encountering the tactical decision point, offers a greater degree of engagement dominance in the earlier portion of the engagement, and hence, the possibility of greater attrition of the adversary.

The former appears to have little or no effect on the measures of effectiveness for small changes in the near-neighborhood of neutral.

CHAPTER 7

SUMMARY

"I know you think you understood what I said, but I am not sure you understand that what I said was not what I meant."
-- Anonymous

THE ANALYSIS APPROACH

It has been shown that for many system interactions, and specifically for the example shown, the deterministic view is unacceptable when compared with experimental results. Statistical descriptors as to the outcome of such interactions, while useful, are less pleasing than statements about the form and substance of the interaction. Details in the form of statistical descriptors of the interaction can be made if the priorities and goals are well enough defined to do merit ordering (the development of a figure of merit). These can further be developed into measures of effectiveness which might describe a group of interactions. Specifically, the following actions must be taken in order to achieve this set of descriptors.

- The definition of the problems in terms of its history, goals, and priorities. (Chapter 2 for the example set.)

- The development of a figure of merit describing these goals and priorities. (Chapter 3 for the example set -- a general definition set for the arbitrary system interaction can be found in Wymore (reference 32) which describes a system-theory analysis of large scale man/machine systems interfaces.)

- The careful definition of statistical tools to be used as descriptors. (Chapter 4 for the example set.)

- The development of measures of effectiveness. (Chapter 5 for the example set.)

FOLLOW ON WORK

The development of analytic tools for the characterization and analysis of experimental data has been shown. While the characterization of experimental data is useful in the analysis of engagement trials, it can also be used in the development of a cause/effect relationship for the extrapolation of data to situations not tested or not testable. Such a predictor model would be "real-world" and involve multi-aircraft air combat as well as the incorporation of actual flight test data which few models presently do. Figure 38 shows the basic construction of such a predictor model as

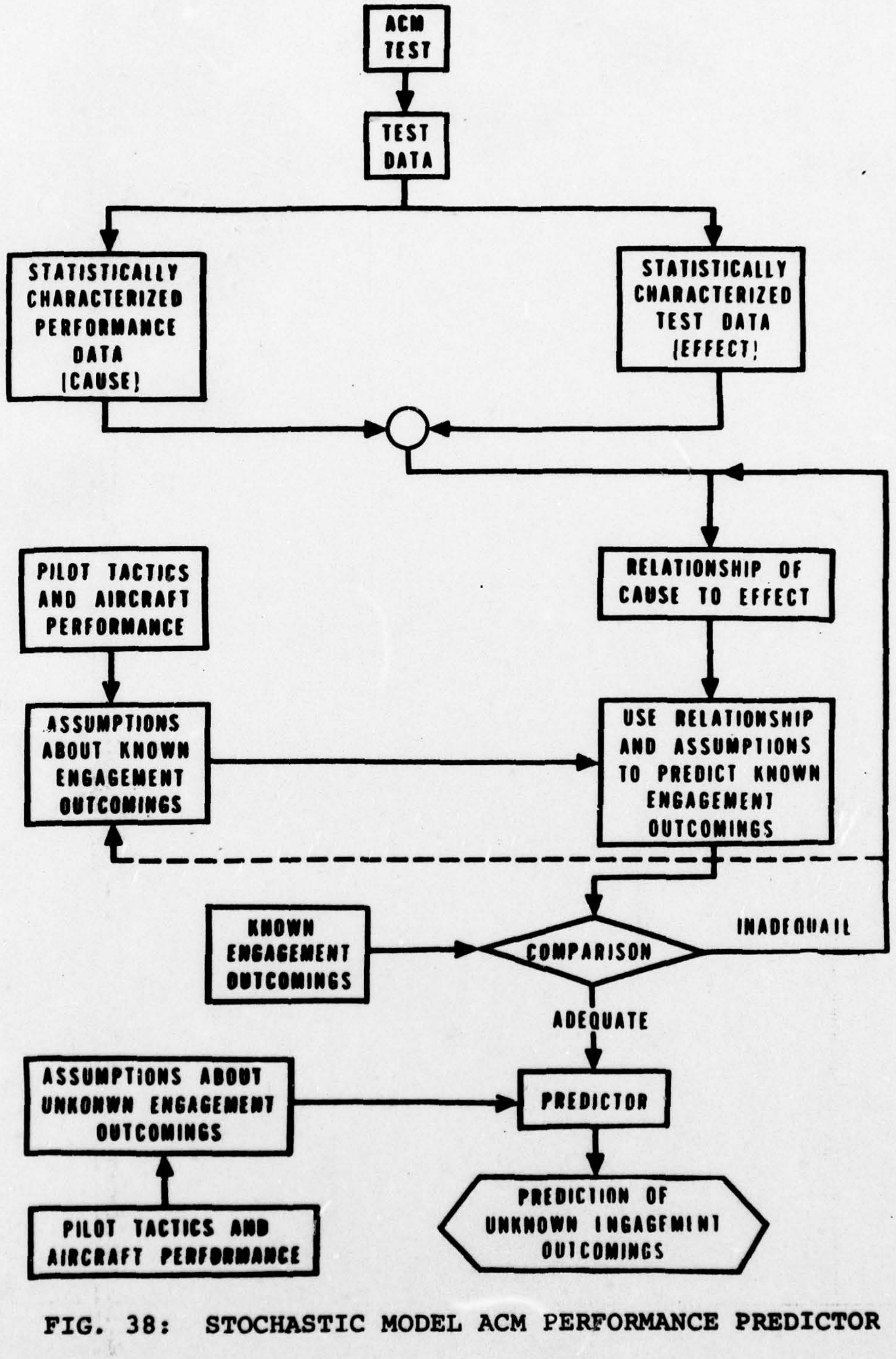


FIG. 38: STOCHASTIC MODEL ACM PERFORMANCE PREDICTOR

applied to the example case under consideration. The main requirement is in the creation of two identically structured, but independently executed data sets. The first of these would be used to create independently generated cause and effect data as shown. These cause and effect data will then be used to generate a mathematical relationship through one of the many available mathematical tools. This relationship, together with the required input assumptions will be used to predict the statistically characterized outcomes of the second data set. The input assumptions and mathematical relations will then be adjusted until adequate prediction is achieved. Table 12 presents the basic types of assumptions required by the proposed model in comparison to present models. The most significant assumptions which are relaxed, are those of the perfect information and perfect pilot as discussed in Chapters 1-3.

AREAS OF APPLICATION

For the example data set, the analysis method has direct application in several areas including; aircraft design, test and evaluation, pilot training and proficiency, force strength projections, etc. Some specific examples follow:

TABLE 12

COMPARISON OF PREDICTOR TYPES
(ASSUMPTIONS REQUIRED)

Digital Simulation (TACTICS II, NORTAC, Decision Sciences, etc.)	Manned Simulation Flight Test (Stochastic Test Model)
1. Deterministic	1. Non-deterministic
2. Finite dimensional	2. Finite dimensional
3. Finite response	3. Infinite response
4. Tactics oriented	4. No operational assumptions required
5. Perfect information	
6. Perfect pilot	

- Pilot Training--Applications to training can be made in the area of multiaircraft fighter section coordination and the identification of critical points in the engagement, as given in Chapter 5. These figures can be used as measures of proficiency for pilot/wingman combinations or fighter squadron readiness as compared to an established criterion.

- Aircraft and Armament Design--The direct application of performance index data to the design problem can be achieved by determining the sensitivity of engagement outcomes to variables under the control of the designer, through either a predictor model or a direct test of concepts.

- Operational Planning--Estimated force strength requirements are a direct fallout of the predictor model as discussed. Air superiority force strengths can be based on realistic projections of fighter aircraft attrition in air combat maneuvering.

- Weapon System Effectiveness--The performance index method can be used as a measure of total system effectiveness in fighter aircraft test and

evaluation. The mission systems effectiveness for the fighter mission would be given as the set of engagement outcomes.

- Airplane System Test and Evaluation--Individual hardware (maneuvering flaps, thrust reversers, etc.) can be tested by its ability to change the performance index distribution in a given set of tests as shown in Chapter 5.

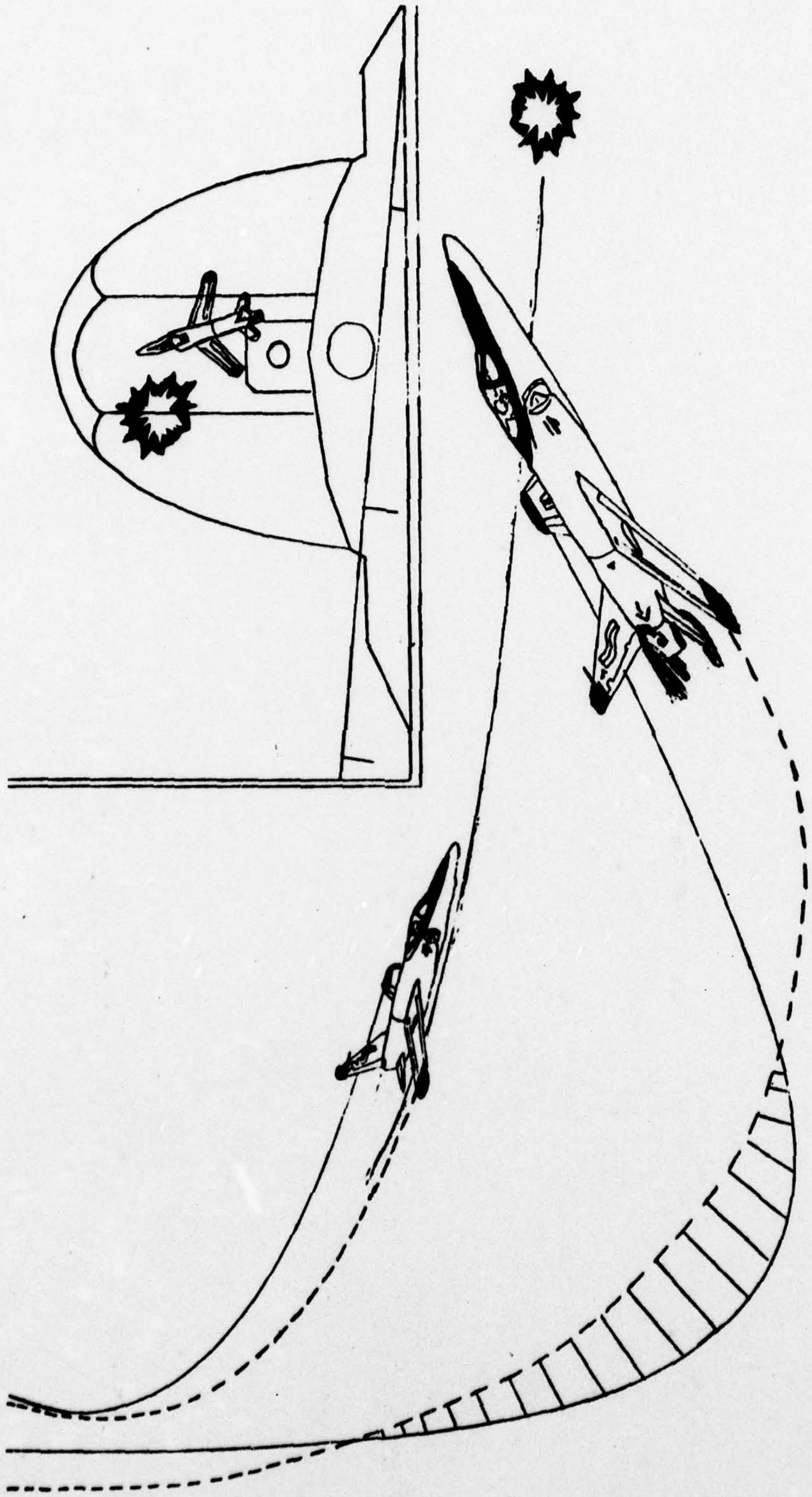


FIG. 39: PARTING SHOT

LIST OF REFERENCES

1. G.E.P. Box and G.M. Jenkins, "Time Series Analysis: Forecasting and Control," Holden-Day, Inc, San Francisco 1976.

2. Bryson, A.E., Jr. and Yu-Chi Ho, "Applied Optimal Control Optimization, Estimation, and Control," J. Wiley & Sons, New York, 1975.

3. Wiener, N., "Cybernetics," The MIT Press, Cambridge, Mass., 1948.

4. Gellman, R., "Adaptive Control Processes: A Guided Tour," Princeton University Press, Princeton, N.J., 1961.

5. W.R. Nunn and R.A. Oberle, "Evaluating Air Combat Maneuvering Engagements," Center for Naval Analyses CNS 1077, Volume 1, September 1976.

6. Anonymous, "The First Air-to-Air Combat," From handout notes at the Naval Fighter Weapons School, NAS, Miramar, Ca., undated.

7. R.L. Spicer and L.G. Martin, "TACTICS II, Maneuver Logic for Computer Simulation of Dogfight Engagements," Rand Corp R-979-PR, Jul 1972.

8. R. Isaacs, "Differential Games," John Wiley and Sons, New York, 1965.

9. Falco, M., "The Analysis of Tactics and System Capability in Aerial Dogfight Game Models," Grumman Aerospace Corporation Report RE-474, May 1974.

10. Peng, W.Y. and Vincent, T.L., "Some Aspects of Aerial Combat," AIAA Journal, Vol. 13, No. 1, January 1975.

11. Northrop Corporation Report NB74-72, "F-5E Combat Tactics Manual," Part 3 Air-to-Air Combat Effectiveness, of Sep 1974.

12. R.A. Oberle, Center for Naval Analyses report No. CRC 274, "Air Combat Maneuver Conversion Model," of Nov 1974.

13. Cubic Corporation Report No. P-74000, "The Air Combat Maneuvering Range (ACMR) An/USQ-T2(V)," 1974.

14. "ACEVAL/AIMVAL Preliminary Test Plan," Volume III (Engineering) FOR OFFICIAL USE ONLY, 22 February 1976.

15. Cubic Corporation Report No. SP/525-5A, "Performance Specification for the Control an Computation Subsystem (CCS)," October 1974.

16. Cubic Corporation Report No. SP/006-201, "Specification for Tracking Instrumentation Subsystem (TIS)," 1974.

17. Cubic Corporation Report No. SP/525-38C, "Specification for Airborne Instrumentation Subsystem (AIS)," October 1974.

18. Simpson, W.R., NavAirTestCen Technical Memorandum TM-76-1SA, "Development of a Time-Variant Figure-of-Merit for Use in the Analysis of Air Combat Maneuvering Engagements," 16 July 1976.

19. Jane's All the World's Aircraft, Various Years, Jane's Yearbooks, London, England.

20. The 1969 Aerospace Yearbook, Aerospace Industries Association, Books, Inc., Washington, D.C., 1969.

21. The F-18 Hornet Strike Fighter, McDonnell Douglas Corporation, Saint Louis, Missouri, 1977.

22. Oberle, R.A., Naron, S.E., and Simpson, W.R., Center for Naval Analyses Report No. CRC 355, "The Air Combat Maneuvering Range Readiness Estimation System," Seven volumes, of November 1978.

23. Youngling, E.W., et. al., McDonnell Douglas Astronautics Co. Report No. MDC E1634, "Feasibility Study to Predict Combat Effectiveness for Selected Military Roles: Fighter Pilot Effectiveness," of April 1977.

24. Simpson, W.R., and Oberle, R.A., NavAirTestCen Technical Memorandum TM-77-2SA, "The Numerical Analysis of Air Combat Engagements Dominated by Maneuvering Performance," 20 June 1977.

25. Mood, A.M., Graybill, F.A., and Boes, D.C., "Introduction to the Theory of Statistics," 3rd Edition, McGraw-Hill, Inc., New York, 1974.

26. VanTrees, H.L., "Detection Estimation and Modulation Theory," John Wiley and Sons, New York, 1968.

27. Curry, R. and Egbert, R., "Investigation of Distribution Recovery Techniques," Operations Research and Economic Analysis Development Department, Ultrasystems, Inc., Newport Beach, Ca., Feb 1974.

28. Parzen, E., "On the Estimation of a Probability Density Function and Mode," Annals of Mathematical Statistics, Vol. 33, pp. 1065-1076, 1962.

29. Murthy, V.K., "Nonparametric Estimation of Multivariate Densities with Applications," Multivariate Analysis (Proceedings of the International Symposium, Dayton, Ohio, 1965), pp 43-56, Academic Press, New York, 1966.

30. Systems 360 Scientific Subroutine Package (360A-CM-03X) Version II Programmers Manual, International Business Machines Report H20-0205-2, White Plains, New York, 1967.

31. Kendall, H.G., "The Advanced Theory of Statistics," Vol. I, Hafner Publishing Company, New York, 1958.

32. Wymore, A.W., "Systems Engineering Methodology for Interdisciplinary Teams," Wiley & Sons, New York, 1976.

APPENDIX A

LIST OF SYMBOLS AND ABBREVIATIONS

APPENDIX A

LIST OF SYMBOLS AND ABBREVIATIONS

a_{ij}	Squared difference term
AA	Angular Advantage
ACE1	Program for Statistical Analysis
ACEVAL	Air Combat Evaluation
ACM	Air Combat Maneuvering
ACMI	Air Combat Maneuvering Instrumentation
ACMR	Air Combat Maneuvering Range
ACMRDB	Driver Program to Produce Cumulative Engagement Tapes From ACMR Tapes
AIS	Airborne Instrumentation Subsystem
ALT	Altitude
AOA	Angle of Attack
AOT	Angle Off Tail
ATA	Antenna Train Analge
BINGO	Low Fuel State, or Prebriefed Fuel for Return to Base
BUGOUT	Engagement Breakoff by One of the Combatant Sections
c	Proportionality Constant
CCS	Computation and Control Subsystem
CRTREC	Control Subprograms for Real-Time Records
δ	Empirically Derived Constant
DA	Directional Angle

DEBRIEF	Subprogram to Read Debrief Data
DDS	Display and Debriefing Subsystem
ENVIND	Subprogram to Compute Envelope Indications
ΔE_s	Differential Energy
e	Natural Constant (2.7183)
E_{dev}	Energy Coefficient
E_s	Specific Energy
$\bar{E}_s$	Average Specific Energy
f	Frequency
FM	Figure of Merit
f_M	Measured (computed) Frequency
f_r	Range Performance Penalty Function
f_{RG}	Interenvelope Gun Penalty
FREDIS	Subprogram for Frequency Computation
FRL	Fuselage Reference Line
f*	Interenvelope Gun Constant
GETOBS	Subprogram to Read ACMR Data Tapes
GCI	Ground Control Intercept
H	Weighting Function
IAP	Subprogram to Compute Interaircraft Parameters
IAS	Indicated Airspeed
$I_{\Delta E_s}$	Differential Energy Integral
Kg	Kilogram
K	Energy Influence Function
K*	Proportionality Constant

lb	Pound
M	Sample Size
MOMEN	Subprogram for Moment Computation
MT	Target Mach Number
N	Newton
N_z	Normal Acceleration (load Factor)
P	Probability
PI	Performance Index
QSF	Subprogram for Integration
R	Interairplane Range
R_G	Range at which gun tactics begin to control the engagement
R_{MAX}	Maximum Range
R_{MIN}	Minimum Range
R_{OPT}	Optimum Range
R_0	Zero Penalty Range
RDEVAL	Subprogram to Read Cumulative Tape File
SAP	Subprogram to Compute Single Aircraft Parameters
START	Subprogram for Determining Engagement Start Points
tally	Visual Sighting of an Opponent
t	Time
TIS	Tracking Instrumentation Subsystem
V	Velocity Vector
VC	Closing Vector
VIFF	Vectoring in Forward Flight

X	Computed Point in Random Field
X_i	Observed Random Data Point
Y_R	Computed Point in Random Field
μ	Relative Sample Size
ρ	Exponential Multiplier (Empirical)
ε	Mean Square Error

<u>Subscripts</u>:

1	Aircraft Pairing 1
2	Aircraft Pairing 2
DEF	Defensive
OFF	Offensive
s	Section
i	Aircraft Pairing i or point i
j	Aircraft Pairing j or point j
Total	Total (time)
N	Normalized

<u>Others</u>:

$x(y)$	x as a function of y
$\langle a,b \rangle$	Interval

APPENDIX B

FIGHTER PERFORMANCE COMPARISON
(References 24-26)

TABLE 13

SIZE[1]

	Wing Span	Length	Weight
MIG-17 FRESCO	32 ft (9.8m)	31 ft (9.5m)	11,000 lb (4 990 Kg)
MIG-19 FARMER	30 ft (9.2m)	34 ft (10.4m)	16,000 lb (7 258 Kg)
MIG-21 FISHBED	24 ft (7.3m)	46 ft (14.0m)	16,500 lb (7 484 Kg)
MIG-23 FLOGGER	26/45 ft[2] (7.9/13.7m)	55 ft (16.8m)	28,000 lb (12 700 Kg)
MIG-25 FOXBAT	44 ft (13.4m)	73 ft (22.3m)	58,000 lb (26 360 Kg)[3]
F-4 PHANTOM	38 ft (11.6m)	58 ft (17.7m)	40,000 lb (18 144 Kg)
F-8 CRUSADER	36 ft (11.0m)	55 ft (16.8m)	25,000 lb (11 340 Kg)
F-14A TOMCAT	38/64 ft[2] (10.1/19.5m)	62 ft (18.9m)	51,000 lb (23 133 Kg)
A-4E SKYHAWK	27 ft (8.2m)	43 ft (13.1m)	12,000 lb (5 443 Kg)
F-5E TIGER	27 ft (8.2m)	49 ft (14.9m)	13,000 lb (5 897 Kg)
F-106 DART	40 ft (12.2m)	70 ft (21.3m)	30,000 lb (13 608 Kg)
F-15 EAGLE	45 ft (13.7m)	64 ft (19.5m)	36,000 lb (16 330 Kg)
F-18 HORNET	40 ft (12.2m)	56 ft (17.1m)	33,000 lb (14 970 Kg)

[1] Figures are approximate and do not account for nose-boom extensions or variations among specific models.

[2] Variable geometry aircraft wings swept/extended.

[3] Estimated.

TABLE 14

WING LOADING[1]

MIG-17 FRESCO	43 lb/ft^2	2058 N/m^2
MIG-19 FARMER	57 lb/ft^2	2729 N/m^2
MIG-21 FISHBED	66 lb/ft^2	3160 N/m^2
MIG-23 FLOGGER	66[2] lb/ft^2	3160 N/m^2
MIG-25 FOXBAT	73[3] lb/ft^2	3495 N/m^2
F-4 PHANTOM	73 lb/ft^2	3495 N/m^2
F-8 CRUSADER	66 lb/ft^2	3160 N/m^2
F-14A TOMCAT	48[2] lb/ft^2	2298 N/m^2
A-4E SKYHAWK	49 lb/ft^2	2346 N/m^2
F-5E TIGER	72 lb/ft^2	3447 N/m^2
F-106 DART	45 lb/ft^2	2154 N/m^2
F-15 EAGLE	54 lb/ft^2	2585 N/m^2
F-18 HORNET	55 lb/ft^2	2633 N/m^2

Notes: (1) At Combat Weight
(2) Wings Forward
(3) Estimated

TABLE 15

THRUST TO WEIGHT[1]

	Uninstalled lb/lb or N/Kg (sea-level mass equiv.)	Installed lb/lb or N/Kg (sea-level mass equiv.)
MIG-17 FRESCO	.71	.66
MIG-19 FARMER	.90	.72
MIG-21 FISHBED	.85	.68
MIG-23 FLOGGER	.98	.85
MIG-25 FOXBAT	Unknown[2]	Unknown[2]
F-4 PHANTOM	.92	.78
F-8 CRUSADER	.78	.68
F-14A TOMCAT	.85	.67
A-4E SKYHAWK	.74	.60
F-5E TIGER	.74	.66
F-106 DART	.80	.62
F-15 EAGLE	1.10	1.00
F-18 HORNET	1.03	.95

Notes: (1) Sea Level Full Afterburner
(2) Unavailable Through Unclassified Sources

TABLE 16

COMBAT FUEL

Fighter	Fuel[1]	Engines	Fuel Flow[3]
MIG-17 FRESCO	2,600 lb (1182 Kg)	1	5.0 lb/sec (2.3 Kg/sec)
MIG-19 FARMER	3,970 lb (1804 Kg)	2	8.3 lb/sec (3.8 Kg/sec)
MIG-21 FISHBED	5,470 lb (2486 Kg)	1	11.7 lb/sec (5.3 Kg/sec)
MIG-23 FLOGGER	9,860 lb (4482 Kg)	1	16.7 lb/sec (7.6 Kg/sec)
MIG-25 FOXBAT	30,800 lb (14000Kg)	2	Unknown[2]
F-4 PHANTOM	13,700 lb (6227 Kg)	2	25.0 lb/sec (11.4 Kg/sec)
F-8 CRUSADER	9,300 lb (4227 Kg)	1	15.0 lb/sec (6.8 Kg/sec)
F-14A TOMCAT	16,400 lb (7454 Kg)	2	33.3+ lb/sec (15.2 Kg/sec)
A-4E SKYHAWK	5,200 lb (2364 Kg)	1	6.7 lb/sec (3.0 Kg/sec)
F-5E TIGER	4,360 lb (1982 Kg)	2	10.0 lb/sec (4.5 Kg/sec)
F-106 DART	11,000 lb (5000 Kg)[4]	1	15.0[4] lb/sec (6.8 Kg/sec)
F-15 EAGLE	10,500 lb (4773 Kg)	2	33.3+ lb/sec (15.2 Kg/sec)
F-18 HORNET	10,400 lb (4727 Kg)	2	30.0 lb/sec (13.6 Kg/sec)

Notes: (1) Internal Fuel Only
(2) Unavailable through unclassified sources
(3) Maximum Afterburner
(4) Estimated

APPENDIX C

SUBROUTINE FOR THE COMPUTATION OF EMPIRICAL DISTRIBUTIONS

SUBROUTINE FOR THE COMPUTATION OF EMPIRICAL DISTRIBUTIONS

```
      SUBROUTINE FREDIS(ALPHA,BETA,DELTA,X,NDATA,FREQ,CUMP,ANS)
      COMMON FREOCC(186,101),XMOM(186,4),SHOT80(190),SHOM80(190)
      COMMON PIS(300,33)
      DIMENSION FREQ(101),CUMP(101),ANS(4)
C-
C-  THIS SUBROUTINE USES FREQUENCY DISTRIBUTION
C-  RECOVERY TECHNIQUE---*** PER ULTRASYSTEMS RPT
C-
C-  IT CALLS AN INTEGRATION SUBROUTINE (QSF) AND A MOMENT CALCULATION
C-  SUBROUTINE (MOMEN)
C-
C-  THE DISTRIBUTION FUNCTION OF THE X STRING VECTOR IS COMPUTED
C-  AND PLACED IN FREQ AFTER IT IS NORMALIZED
C--
C-  X HAS NDATA VALUES WITH A MAXIMUM OF BETA (RANGE VALUE)
C-  A MINIMUM OF ALPHA (RANGE VALUE) TO BE COMPUTED OVER
C-  A GRID VALUE OF DELTA
C-
C-  THE PROGRAM ALSO RETURNS THE NORMALIZED CUMULATIVE PROBABILITY
C-  OF OCCURENCE (CUMP) AND THE FIRST FOUR MOMENTS (ANS)
C-
      DIMENSION UBO(5),X(1),Y(186),XI(186)
      UBO(1)=ALPHA
      UBO(2)=BETA
      UBO(3)=DELTA
      SUM1=0.
      SUM2=0.
      NPTS=(BETA-ALPHA)/DELTA+1.25
      SUMAIJ=0.
      NINT=(BETA-ALPHA)/DELTA
      DO 10 I=1,NDATA
      DO 10 J=1,NDATA
   10 SUMAIJ=(X(I)-X(J))**2+SUMAIJ
      XMU=0.
      XMU=SUMAIJ
      XMU=XMU/(NDATA*(NDATA-1))
      IF(ABS(XMU).LT.1.E-08) RHO=0.
      IF(ABS(XMU).LT.1.E-08) GO TO 22
      IF(XMU.GT.1.) DELTA1=0.010
      XDATA=NDATA
      IF(XMU.GT.1.) RHO=(DELTA1*ALOG(XDATA)+ALOG(XMU))/XMU
      IF(XMU.LE.1) DELTA1=0.250
      IF(XMU.LE.1) RHO=DELTA1*ALOG(XDATA)/XMU
```

```
   22 CONTINUE
      BINV=0.
      DO 30 I=1,NDATA
      DO 30 J=1,NDATA
      IF((RHO*(X(I)-X(J))**2).GT.80.) ECH=0.
      IF((RHO*(X(I)-X(J))**2).GT.80.) GO TO 23
      ECH=EXP(-1.*RHO*(X(I)-X(J))**2)
   23 FUNC=(X(I)-X(J))**2*ECH
   30 BINV=BINV+FUNC
      IF(ABS(BINV).LT.1.E-08) B=0.
      IF(ABS(BINV).LT.1.E-08) GO TO 32
      BINV=BINV/(NDATA*(NDATA-1))
      B=1./BINV
   32 CONTINUE
      DO 40 I=1,NPTS
   40 Y(I)=ALPHA+(I-1)*DELTA
      DO 50 I=1,NPTS
      FREQ(I)=0.
      DO 50 J=1,NDATA
      STEP1=Y(I)-X(J)
      IF(ABS(STEP1*B).GT.12.) STEP2=0.0
      IF(ABS(STEP1*B).GT.12.) GO TO 50
      STEP2=EXP(-1*(B*STEP1)**2/2.)
   50 FREQ(I)=FREQ(I)+B/NDATA*STEP2
      CALL QSF(DELTA,FREQ,XI,NPTS)
      DO 500 J=1,NPTS
      IF(ABS(XI(NPTS)).GT.1.E-08) GO TO 490
      FREQ(J)=0.
      CUMP(J)=0.
      GO TO 500
  490 FREQ(J)=FREQ(J)/XI(NPTS)
      CUMP(J)=XI(J)/XI(NPTS)
  500 CONTINUE
      CALL MOMEN(FREQ,UBO,NPTS,ANS)
      RETURN
      END
```

SUBROUTINE FOR THE COMPUTATION OF INTEGRALS (Evenly Spaced)

```
      SUBROUTINE QSF(H,Y,Z,NDIM)
      COMMON FREQCC(186,101),XMOM(186,4),SHOT80(190),SHOM80(190)
      COMMON PIS(300,33)
C-
C-    THIS IS A GENERAL INTEGRATION ROUTINE FOR EQUALLY SPACED
C-     FUNCTIONS (Y)
C-
C-  THE SPACING OF Y IS H    THE NUMBER OF POINTS IS NDIM
C-   THE INTEGRAL OF Y IS Z
C-
C-
C-   THE SUBROUTINE USES A COMBINATION OF SIMPSONS RULE
C-  AND NEWTONS THREE-EIGTH RULE
C-
C-
      DIMENSION Y(1),Z(1)
      HT=H*1./3.
    1 SUM1=Y(2)+Y(2)
      SUM1=SUM1+SUM1
      SUM1=HT*(Y(1)+SUM1+Y(3))
      AUX1=Y(4)+Y(4)
      AUX1=AUX1+AUX1
      AUX1=SUM1+HT*(Y(3)+AUX1+Y(5))
      AUX2=HT*(Y(1)+3.875*(Y(2)+Y(5))+2.625*(Y(3)+Y(4))+Y(6))
      SUM2=Y(5)+Y(5)
      SUM2=SUM2+SUM2
      SUM2=AUX2-HT*(Y(4)+SUM2+Y(6))
      Z(1)=0.
      AUX=Y(3)+Y(3)
      AUX=AUX+AUX
      Z(2)=SUM2-HT*(Y(2)+AUX+Y(4))
      Z(3)=SUM1
      Z(4)=SUM2
      IF(NDIM-6) 5,5,2
    2 DO 4 I=7,NDIM,2
      SUM1=AUX1
      SUM2=AUX2
      AUX1=Y(I-1)+Y(I-1)
      AUX1=AUX1+AUX1
      AUX1=SUM1+HT*(Y(I-1)+AUX1+Y(I))
      Z(I-2)=SUM1
      IF(I-NDIM) 3,6,6
    3 AUX2=Y(I)+Y(I)
      AUX2=AUX2+AUX2
```

```
      AUX2=SUM2+HT*(Y(I-1)+AUX2+Y(I+1))
    4 Z(I-1)=SUM2
    5 Z(NDIM-1)=AUX1
      Z(NDIM)=AUX2
      RETURN
    6 Z(NDIM-1)=SUM2
      Z(NDIM)=AUX1
      RETURN
      END
```

SUBROUTINE FOR THE COMPUTATION OF CLASSICAL MOMENTS (1 to 4)

```
      SUBROUTINE MOMEN(F,UBO,NPTS,ANS)
      COMMON FREOCC(186,101),XMOM(186,4),SHOT80(190),SHOM80(190)
      COMMON PIS(300,33)
      DIMENSION ANS(4)
C-
C-  THIS ROUTINE CALCULATES THE CLASSICAL MOMENTS OF A GIVEN
C-  FREQUENCY FUNCTION (F) HAVING A MAX (UBO(2)) A MIN (UBO(1))
C-  AND A GRID SIZE(UBO(3)) THE MOMENTS ARE PLACED IN ANS
C-
C-  THE FIRST MOMENT (MEAN IS ABOUT THE ORIGIN)
C-  ALL SUBSEQUENT MOMENTS ARE ABOUT THE MEAN
C-
      DIMENSION F(1),XM(186),Y(186)
      DIMENSION UBO(3)
      DO 10 I=1,4
   10 ANS(I)=0.
      DO 20 I=1,NPTS
      Y(I)=UBO(1)+(I-1)*UBO(3)
   20 XM(I)=Y(I)*F(I)
      CALL QSF(UBO(3),XM,XM,NPTS)
      ANS(1)=XM(NPTS)
      DO 30 I=1,NPTS
   30 XM(I)=F(I)*(Y(I)-ANS(1))**2
      CALL QSF(UBO(3),XM,XM,NPTS)
      ANS(2)=XM(NPTS)
      DO 40 I=1,NPTS
   40 XM(I)=F(I)*(Y(I)-ANS(1))**3
      CALL QSF(UBO(3),XM,XM,NPTS)
      ANS(3)=XM(NPTS)
      DO 50 I=1,NPTS
   50 XM(I)=F(I)*(Y(I)-ANS(1))**4
      CALL QSF(UBO(3),XM,XM,NPTS)
      ANS(4)=XM(NPTS)
      RETURN
      END
```